

Additional Mathematics- 0606**Series questions (new topic – various styles of questions for practice)**

These questions are hard in general so if you can solve them then your concepts are clear!

1)

The third term of an arithmetic series is 70 and the sum of the first 10 terms of the series is 450

(a) Calculate the common difference of the series.

(4)

The sum of the first n terms of the series is S_n

Given that $S_n \geq 350$

(b) find the set of possible values of n .

(6)

2)

The ninth term of an arithmetic progression is 22 and the sum of the first 4 terms is 49.

(i) Find the first term of the progression and the common difference.

[4]

The n th term of the progression is 46.

(ii) Find the value of n .

[2]

3)

The sum of the first and third terms of a geometric series is 100

The sum of the second and third terms is 60

(a) Find the two possible values of the common ratio of the series.

(5)

Given that the series is convergent, find

(b) the first term of the series,

(2)

(c) the least number of terms for which the sum is greater than 159.9

(4)

4)

An arithmetic series has first term a and common difference d . The n th term of the series is t_n and the sum of the first n terms of the series is S_n

(a) Write down an expression in terms of a and d for

(i) t_{58}

(ii) S_{13}

(2)

Given that $t_{58} = S_{13}$

(b) show that $d = -\frac{4}{7}a$

(2)

(c) show that $t_{176} = S_{21}$

(4)

(d) find the value of r when $t_r = 5t_9$

(3)

5)

(a) An arithmetic progression contains 25 terms and the first term is -15 . The sum of all the terms in the progression is 525. Calculate

(i) the common difference of the progression, [2]

(ii) the last term in the progression, [2]

(iii) the sum of all the positive terms in the progression. [2]

(b) A college agrees a sponsorship deal in which grants will be received each year for sports equipment. This grant will be \$4000 in 2012 and will increase by 5% each year. Calculate

(i) the value of the grant in 2022, [2]

(ii) the total amount the college will receive in the years 2012 to 2022 inclusive. [2]

6)

The first term of an arithmetic progression is 12 and the sum of the first 9 terms is 135.

(i) Find the common difference of the progression. [2]

The first term, the ninth term and the n th term of this arithmetic progression are the first term, the second term and the third term respectively of a geometric progression.

(ii) Find the common ratio of the geometric progression and the value of n . [5]

7)

The first term of an arithmetic progression is 61 and the second term is 57. The sum of the first n terms is n . Find the value of the positive integer n . [4]

8)

- (a) The first two terms of an arithmetic progression are 16 and 24. Find the least number of terms of the progression which must be taken for their sum to exceed 20 000. [4]
- (b) A geometric progression has a first term of 6 and a sum to infinity of 18. A new geometric progression is formed by squaring each of the terms of the original progression. Find the sum to infinity of the new progression. [4]

9)

- (a) A geometric progression has first term $3a$ and common ratio r . A second geometric progression has first term a and common ratio $-2r$. The two progressions have the same sum to infinity. Find the value of r . [3]
- (b) The first two terms of an arithmetic progression are 15 and 19 respectively. The first two terms of a second arithmetic progression are 420 and 415 respectively. The two progressions have the same sum of the first n terms. Find the value of n . [3]

10)

- (a) Two convergent geometric progressions, P and Q , have the same sum to infinity. The first and second terms of P are 6 and $6r$ respectively. The first and second terms of Q are 12 and $-12r$ respectively. Find the value of the common sum to infinity. [3]
- (b) The first term of an arithmetic progression is $\cos \theta$ and the second term is $\cos \theta + \sin^2 \theta$, where $0 \leq \theta \leq \pi$. The sum of the first 13 terms is 52. Find the possible values of θ . [5]

P.T.O

11)

A water tank holds 2000 litres when full. A small hole in the base is gradually getting bigger so that each day a greater amount of water is lost.

- (i) On the first day after filling, 10 litres of water are lost and this increases by 2 litres each day.
- (a) How many litres will be lost on the 30th day after filling? [2]
- (b) The tank becomes empty during the n th day after filling. Find the value of n . [3]
- (ii) Assume instead that 10 litres of water are lost on the first day and that the amount of water lost increases by 10% on each succeeding day. Find what percentage of the original 2000 litres is left in the tank at the end of the 30th day after filling. [4]

12)

The first term of a geometric series is 5 and the common ratio is 1.2

For this series find, to 1 decimal place,

- (a) (i) the 20th term,
 (ii) the sum of the first 20 terms. (4)

The sum of the first n terms of the series is greater than 3000

- (b) Calculate the smallest possible value of n . (4)

13)

- (i) All the terms of a geometric series are positive. The sum of the first two terms is 34 and the sum to infinity is 162

Find

- (a) the common ratio, (4)
 (b) the first term. (2)

- (ii) A different geometric series has a first term of 42 and a common ratio of $\frac{6}{7}$.

Find the smallest value of n for which the sum of the first n terms of the series exceeds 290

(4)

14)

The first term of a geometric series is 6 and the common ratio is 0.92

For this series, find

- (a) (i) the 25th term, giving your answer to 2 significant figures,
 (ii) the sum to infinity.

(4)

The sum to n terms of this series is greater than 72

- (b) Calculate the smallest possible value of n .

(4)**15)**

An arithmetic sequence, u_1, u_2, u_3, \dots , has $d = 11$ and $u_{27} = 263$.

- (a) Find u_1 . [2 marks]

- (b) (i) Given that $u_n = 516$, find the value of n .

- (ii) For this value of n , find S_n . [4 marks]
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16)

- (a) A geometric progression has a third term of 20 and a sum to infinity which is three times the first term. Find the first term. [4]

- (b) An arithmetic progression is such that the eighth term is three times the third term. Show that the sum of the first eight terms is four times the sum of the first four terms. [4]
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17)

Evaluate $\sum_{r=5}^{60} (2r + 7)$

(4)

18)

The sum of the first term and the third term of a geometric series is 75

The sum of the second term and the third term is 45

(a) Find the two possible values of the common ratio of the series.

(5)

Given that the series is convergent with sum to infinity S ,

(b) find the value of S .

(3)

19)

A geometric series has first term $(11x - 3)$, second term $(5x + 3)$ and third term $(3x - 3)$.

(a) Find the two possible values of x .

(4)

For each of your values of x ,

(b) find the corresponding value of the common ratio of the series.

(3)

Given that the series is convergent,

(c) find the sum to infinity of the series.

(3)

20)

(a) The fifth term of an arithmetic progression is 18 and the sum of the first 5 terms is 75. Find the first term and the common difference. [4]

(b) The first term of a geometric progression is 16 and the fourth term is $\frac{27}{4}$. Find the sum to infinity of the progression. [3]

21)

(a) The fifth term of an arithmetic progression is 18 and the sum of the first 5 terms is 75. Find the first term and the common difference. [4]

(b) The first term of a geometric progression is 16 and the fourth term is $\frac{27}{4}$. Find the sum to infinity of the progression. [3]

22)

The sums of n terms of two arithmetic progressions are in the ratio $5n + 4 : 9n + 6$. Find the ratio of their 18th terms.

23)

The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

24)

Consider the arithmetic sequence 8, 26, 44,

- (a) Find an expression for the n^{th} term. *[1 mark]*
- (b) Write down the sum of the first n terms using sigma notation. *[1 mark]*
- (c) Calculate the sum of the first 15 terms. *[2 marks]*
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25)

The sum of the first 16 terms of an arithmetic sequence is 212 and the fifth term is 8.

- (a) Find the first term and the common difference. *[4 marks]*
- (b) Find the smallest value of n such that the sum of the first n terms is greater than 600. *[3 marks]*
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P.T.O

26)

Phil takes out a bank loan of \$150 000 to buy a house, at an annual interest rate of 3.5%. The interest is calculated at the end of each year and added to the amount outstanding.

- (a) Find the amount Phil would owe the bank after 20 years. Give your answer to the nearest dollar. [3]

To pay off the loan, Phil makes annual deposits of $\$P$ at the end of every year in a savings account, paying an annual interest rate of 2%. He makes his first deposit at the end of the first year after taking out the loan.

- (b) Show that the total value of Phil's savings after 20 years is $\frac{(1.02^{20} - 1)P}{(1.02 - 1)}$. [3]

- (c) Given that Phil's aim is to own the house after 20 years, find the value for P to the nearest dollar. [3]

David visits a different bank and makes a single deposit of $\$Q$, the annual interest rate being 2.8%.

27)

The first term of an infinite geometric sequence is 4. The sum of the infinite sequence is 200.

- (a) Find the common ratio. [2]
- (b) Find the sum of the first 8 terms. [2]
- (c) Find the least value of n for which $S_n > 163$. [3]
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28)

[Maximum mark: 6]

In an arithmetic sequence, $u_2 = 5$ and $u_3 = 11$.

- (a) Find the common difference. [2]
- (b) Find the first term. [2]
- (c) Find the sum of the first 20 terms. [2]
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29)

- (a) A geometric progression has first term 100 and sum to infinity 2000. Find the second term. [3]
- (b) An arithmetic progression has third term 90 and fifth term 80.
- (i) Find the first term and the common difference. [2]
- (ii) Find the value of m given that the sum of the first m terms is equal to the sum of the first $(m + 1)$ terms. [2]
- (iii) Find the value of n given that the sum of the first n terms is zero. [2]
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30)

In an arithmetic progression the first term is a and the common difference is 3. The n th term is 94 and the sum of the first n terms is 1420. Find n and a . [6]
