Additional Mathematics- 0606

Series- Mark Scheme

1)

(a) $a + 2d = 70$

$\frac{10}{2}(2a + 9d) = 450$

$2a + 9d = 90$

$5d = -50 \Rightarrow d = -10$

(b) $a = 70 + 20 = 90$

$S = \frac{n}{2}(180 - 10(n-1))$

$\frac{n}{2}(190 - 10n) \geqslant 350 \qquad 190n - 10n^2 \geqslant 700$

$n^2 - 19n + 70 \leqslant 0$

$(n-5)(n-14) \leqslant 0$

critical values: 5,14

$5 \leqslant n \leqslant 14 \quad n \in \mathbb{Z} \quad (n = 5, 7, \ldots, 13, 14)$

2)

9^{th} term = 22, $S_4 = 49$		
(i) $a + 8d = 22$	B1	co
$2(2a + 3d) = 49$	B1	co
Soln of sim eqns	M1 A1	Solution of two linear sim eqns. co
$\rightarrow d = 1.5, a = 10$	[4]	
(ii) $a + (n-1)d = 46$		
Substitutes for a and d	M1	Correct formula needed and attempt to solve. co.
$\rightarrow n = 25$	A1 [2]	

3)

(a) $a + ar^2 = 100, \quad ar + ar^2 = 60$

$$\frac{1+r^2}{r+r^2} = \frac{100}{60}$$

$$6 + 6r^2 = 10r + 10r^2 \quad 2r^2 + 5r - 3 = 0$$

$$(2r-1)(r+3) = 0$$

$$r = \tfrac{1}{2} \quad r = -3$$

(b) $r = \tfrac{1}{2} \quad a = \dfrac{100}{1+\left(\tfrac{1}{2}\right)^2} = 80$

(c) $S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{80\left(1-\left(\dfrac{1}{2}\right)^n\right)}{1-\dfrac{1}{2}} > 159.9$

$$\frac{159.9}{160} < 1 - \left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{2}\right)^n = 1 - \frac{159.9}{160}$$

$$n \log 0.5 < \log\left(1 - \frac{159.9}{160}\right)$$

$$n > \frac{\log\left(1 - \frac{159.9}{160}\right)}{\log 0.5} = 10.6$$

n=11

P.T.O

4)

(a)	(i) $t_{58} = a + 57d$
	(ii) $S_{13} = \frac{13}{2}(2a + 12d)$
(b)	$a + 57d = \frac{13}{2}(2a + 12d)$
	$-12a = 21d$
	$d = -\frac{4}{7}a$
(c)	$t_{176} = a + 175d = a + 175(-\frac{4}{7}a)$ OR
	$S_{21} = \frac{21}{2}(2a + 20d) = 21a + 210(-\frac{4}{7}a)$
	$= a - 100a = -99a$
	$S_{21} = \frac{21}{2}(2a + 20d) = 21a + 210(-\frac{4}{7}a)$ OR
	$t_{176} = a + 175d = a + 175(-\frac{4}{7}a)$
	$= 21a - 120a = -99a = t_{176}$
(d)	$a + (r-1)d = 5(a + 8d)$
	$(r-1)d = 4(-\frac{7}{4}d) + 40d$ or $(r-1)(-\frac{4}{7}a) = 4a + 40(-\frac{4}{7}a)$
	$r - 1 = 33$ or $-4(r-1) = -132$
	$r = 34$

5)

(a) $a = -15, \quad n = 25$		
(i) Use of $S_n \rightarrow d = 3$.	M1 A1 [2]	Must be correct formula. co
(ii) Last term $= a + 24d$ $\rightarrow 57$ (or $525 = \frac{1}{2} \times 25 \times (-15 + l) \rightarrow l = 57$)	M1 A1√ [2]	Must be $a + 24d$ √ for his d.
(iii) Positive terms are 3,6, ...57 Either $a = 0$ or 3, $n = 19$ or 20	M1	Correct use of formula for S_n.
Use of S_{19} or S_{20} $\rightarrow 570$	A1 [2]	co
(b) $r = 1.05$	B1	In either part **(i)** or **(ii)**.
(i) 11^{th} term $= ar^{10}$ = \$6516 or \$6520	B1 [2]	co
(ii) $S_{11} = \dfrac{4000 \times (1.05^{11} - 1)}{.05}$	M1	Correct sum formula with their r.
= \$56800 or (56827)	A1 [2]	co

6)

(i) Uses S_n $\frac{9}{2}(24+8d)=135 \rightarrow d=\frac{3}{4}$	M1 A1 [2]	Uses correct formula co
(ii) 9^{th} term of AP = $12+8\times\frac{3}{4}=18$ GP 1^{st} tern 12, 2^{nd} term 18 Common ratio = $r = 18 \div 12 = 1\frac{1}{2}$ 3^{rd} term of GP = $ar^2 = 27$ nth term of AP is $12+(n-1)\frac{3}{4}$ $12+(n-1)\frac{3}{4}=27 \rightarrow n=21$	B1√ M1 M1 M1A1 [5]	√ on "d" Uses "ar" Uses ar^2 or "ar" × r Links AP with GP. co

7)

$\frac{n}{2[122+(n-1)(-4)]}$ $n=\frac{n}{2[122+(n-1)(-4)]}$ $2n(n-31)=0$ $n=31$	**M1** **A1** **DM1** **A1** **[4]**	Attempt sum formula with $a = 61$, $d = -4$ Equated to n cao Attempt to solve. Accept div. by n cao

8)

(If answer in decimal then input the exact whole number and a number above it to check if it satisfies, here it is 70 and rounded up)

(a)	$(S_n =)\ \frac{n}{2}[32+(n-1)8]$ and 20000	**M1**
		A1
	$\rightarrow n^2 + 3n - 5000$ (<,=,> 0)	**DM1**
	$\rightarrow (n = 69.2) \rightarrow$ 70 terms needed.	**A1**
	Total:	**4**
(b)	$a = 6,\ \frac{a}{1-r} = 18 \rightarrow r = \frac{2}{3}$	**M1A1**
	New progression $a = 36$, $r = \frac{4}{9}$ oe	**M1**
	New $S_\infty = \frac{36}{1-\frac{4}{9}} \rightarrow 64.8$ or $\frac{324}{5}$ oe	**A1**
	Total:	**4**

9)

(i)	$\dfrac{3a}{1-r}=\dfrac{a}{1+2r}$	M1	Attempt to equate 2 sums to infinity. At least one correct
	$3+6r=1-r$	DM1	Elimination of 1 variable (a) at any stage and multiplication
	$r=-\dfrac{2}{7}$	A1	
		3	
(ii)	$\frac{1}{2}n\left[2\times15+(n-1)4\right]=\frac{1}{2}n\left[2\times420+(n-1)(-5)\right]$	M1A1	Attempt to equate 2 sum to n terms, at least one correct (M1). Both correct (A1)
	$n=91$	A1	
		3	

10)

(a)	$\dfrac{6}{1-r}=\dfrac{12}{1+r}$ $r=\dfrac{1}{3}$ $S=9$	M1 A1 A1	[3]	
(b)	$\dfrac{13}{2}\left[2\cos\theta+12\sin^2\theta\right]=52$	M1*		Use of correct formula for sum of AP
	$2\cos\theta+12(1-\cos^2\theta)=8\to6\cos^2\theta-\cos\theta-2(=0)$	DM1		Use $s^2=1-c^2$ & simplify to 3-term quad
	$\cos\theta=2/3$ or $-1/2$ soi	A1		
	$\theta=0.841$, 2.09 Dep on previous A1	A1A1	[5]	Accept 0.268π, $2\pi/3$. SRA1 for 48.2°, 120° Extra solutions in range –1

P.T.O

11)

(i) (a)	$a+(n-1)d = 10 + 29\times2$	**M1**	Use of *n*th term of an AP with a=±10, d=±2, n=30 or 29
	$= 68$	**A1** [2]	Condone – 68 → 68
(b)	$\frac{1}{2}n(20 + 2(n-1)) = 2000$ or 0	**M1**	Use of S_n formula for an AP with a=±10, d=±2 and equated to either 0 or 2000.
	$\rightarrow 2n^2 + 18n - 4000 = 0$ oe	**A1**	Correct 3 term quadratic = 0.
	(n=) 41	**A1** [3]	
(ii)	$r = 1.1$, oe	**B1**	e.g. $\frac{11}{10}$, 110%
	Uses $S_{30} = \frac{10(1.1^{30}-1)}{1.1-1}$ (= 1645)	**M1**	Use of S_n formula for a GP, a=±10, n=30.
	Percentage lost = $\frac{2000-1645}{2000}\times100$	**DM1**	Fully correct method for % left with "their 1645"
	= 17.75	**A1** [4]	allow 17.7 or 17.8.

12)

(a)(i)	$t_{20} = 5\times1.2^{19} = 159.7$	M1: Use of $t_n = ar^{n-1}$
		A1: Cao
(ii)	$S_{20} = \frac{5(1-1.2^{20})}{1-1.2} = 933.4$	M1: Use of a correct sum formula with n = 19 or n = 20 NB if n = 19 is used and no formula is quoted, score M0
		A1: Cao
(b)	$\frac{5(1-1.2^n)}{1-1.2}(>\text{or}=)3000$	Correct statement (allow 'a' and/or 'r' instead of 5 and 1.2)
	$1.2^n > 121$	1.2^n (> or < or =) k
	$\log1.2^n > \log121$ or $n > \log_{1.2}121$	Takes logs correctly
	$n > \frac{\log121}{\log1.2}$ i.e. $n = 27$	cao
	Ignore symbols e.g. '=' throughout with no errors getting n = 27 scores full marks	
	In (b) Treat $5\times1.2^{n-1} > 3000$ as a misread and allow the M's if scored (gives n = 37)	

13)

.(i)	**Mark (a) and (b) together**
(a)	$a+ar=34$ or $\frac{a(1-r^2)}{(1-r)}=34$ or $\frac{a(r^2-1)}{(r-1)}=34$; $\frac{a}{1-r}=162$
(Way 1)	Eliminate a to give $(1+r)(1-r)=\frac{17}{81}$ or $1-r^2=\frac{34}{162}$.. (not a cubic)
	(and so $r^2=\frac{64}{81}$ and) $r=\frac{8}{9}$ only
(b)	Substitute their $r=\frac{8}{9}$ ($0<r<1$) to give $a=$ $a=18$
(Way 2) Part (b) first	Eliminate r to give $\frac{34-a}{a}=1-\frac{a}{162}$
	gives $a=18$ or 306 and rejects 306 to give $a=18$
Then part (a) again	Substitute a = 18 to give $r=$
	$r=\frac{8}{9}$
(ii)	$\frac{42(1-\frac{6}{7}^n)}{1-\frac{6}{7}}>290$ (For trial and improvement approach see notes below)
	to obtain So $(\frac{6}{7})^n<(\frac{4}{294})$ or equivalent e.g. $(\frac{7}{6})^n>(\frac{294}{4})$ or $(\frac{6}{7})^n<(\frac{2}{147})$
	So $n>\frac{\log"(\frac{4}{294})"}{\log(\frac{6}{7})}$ or $\log_{\frac{6}{7}}"(\frac{4}{294})"$ or equivalent but must be log of positive quantity
	(i.e. $n>27.9$) so $n=28$

14)

(a)(i)	Attempts to use $u_n=ar^{n-1}\Rightarrow u_{25}=6\times0.92^{24}=$ awrt 0.81	M1A1
(ii)	Attempts to use $S_\infty=\frac{a}{1-r}\Rightarrow S_\infty=\frac{6}{1-0.92}=75$	M1A1
		(4)
(b)	Sets $S_n>72\Rightarrow\frac{6(1-0.92^n)}{1-0.92}>72$ Accept $\frac{6(1-0.92^n)}{1-0.92}=72$	M1
	$0.92^n<0.04$ Accept $0.92^n=0.04$	A1
	Takes log's $n>\frac{\log0.04}{\log0.92}$ Accept $n=\frac{\log0.04}{\log0.92}$	dM1
	n=39	A1
		(4)

P.T.O

15)

(a) evidence of equation for u_{27}

e.g. $263 = u_1 + 26\times 11,\ u_{27} = u_1 + (n-1)\times 11,\ 263-(11\times 26)$

$u_1 = -23$

(b) (i) correct equation

e.g. $516 = -23 + (n-1)\times 11,\ 539 = (n-1)\times 11$

$n = 50$

(ii) correct substitution into sum formula

e.g. $S_{50} = \dfrac{50(-23+516)}{2},\ S_{50} = \dfrac{50\left(2\times(-23)+49\times 11\right)}{2}$

$S_{50} = 12325$ (accept 12300)

16)

(a) $ar^2 = 20$	B1	co
$\dfrac{a}{1-r} = 3a$	B1	co
Soln of equations → (r = ⅔) $a = 45$	M1 A1 [4]	Complete method to find *a*. co
(b) $a + 7d = 3(a + 2d)$	M1	Use of $a + (n-1)d$
→ $2a = d$	A1	co
$S_8 = 4(2a + 7d) = 32d$ or $64a$	M1	correct use of S_n formula once.
$S_4 = 2(2a + 3d) = 8d$ or $16a$	A1 [4]	ag

17)

$$\sum_{r=5}^{60}(2r+7)$$

$$=\frac{56}{2}(17+127) \quad \text{or} \quad =\frac{56}{2}(34+55\times 2)$$

$$= 4032$$

Alternative:

$$\sum_{r=5}^{60}(2r+7)=\sum_{1}^{60}(2r+7)-\sum_{1}^{4}(2r+7)$$

$$=\frac{60}{2}(9+127)-\frac{4}{2}(9+15)$$

$$= 4032$$

18)

(a)	$a+ar^2=75$	M1
	$ar+ar^2=45$	A1
	$\frac{1+r^2}{r+r^2}=\frac{75}{45}\quad\left(=\frac{5}{3}\right)$	dM1
	$2r^2+5r-3=0 \qquad (2r-1)(r+3)=0$	
	$r=\frac{1}{2}$ or -3	M1 (NB A1 on e-PEN) A1 (5)
(b)	$a=\frac{75}{\left(1+\frac{1}{4}\right)}=60$	B1
	$S=\frac{a}{1-r}=\frac{60}{\frac{1}{2}}=120 \quad \left(\text{or } S=\frac{a(1-r^n)}{1-r} \text{ with } n=\infty\right)$	M1A1cao (3)
		[8]

19)

(a)	$\frac{(5x+3)}{(11x-3)}=\frac{(3x-3)}{(5x+3)}$ or $(5x+3)^2=(3x-3)(11x-3)$	M1A1
	$25x^2+30x+9=33x^2-42x+9$	
	$8x^2-72x(=0)$ $x=0,\ x=9$	dM1A1 (4)
	Spec case: Give M1A0M0A0 (ie B1) if $x = 0$ seen w/o working	
(b)	$x=0$ $r=\frac{3}{-3}=-1$	B1
	$x=9$ $r=\frac{48}{96}=\frac{1}{2}$	M1A1cso (3)
(c)	$x=9$ $a=96$	
	$S_\infty=\frac{96}{1-\frac{1}{2}},=192$	M1Aft, A1cao (3)
		[10]

20)

(a) $a+4d=18$	B1		co or $75 = 5/2(a + 18) \rightarrow a = 12$ etc
$\frac{5}{2}(2a+4d)=75$	B1		co
Solution	M1		Solution of sim equations
$\rightarrow a = 12,\ d = 1½$	A1	[4]	co for both
(b) $a=16$ and $ar^3=\frac{27}{4}$	B1		Needs both of these
$r=\frac{3}{4}$			
Sum to infinity = 64	M1 A1	[3]	Correct formula and $\|r\|<1$

21)

(a) $a+4d=18$	B1		co or $75 = 5/2(a + 18) \rightarrow a = 12$ etc
$\frac{5}{2}(2a+4d)=75$	B1		co
Solution	M1		Solution of sim equations
$\rightarrow a = 12,\ d = 1½$	A1	[4]	co for both
(b) $a=16$ and $ar^3=\frac{27}{4}$	B1		Needs both of these
$r=\frac{3}{4}$			
Sum to infinity = 64	M1 A1	[3]	Correct formula and $\|r\|<1$

22)

Answer. Let $a_1, a_2, d_1 d_2$ be the first terms and the common difference of the first and second arithmetic progression respectively.
According to the given condition,

$$\frac{\text{Sum of } n \text{ terms of first A.P.}}{\text{Sum of } n \text{ terms of second A.P.}} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{5n+4}{9n+6} \quad \ldots(1)$$

Substituting n = 35 in (1), we obtain

$$\frac{2a_1+34d_1}{2a_2+34d_2} = \frac{5(35)+4}{9(35)+6}$$

$$\Rightarrow \frac{a_1+17d_1}{a_2+17d_2} = \frac{179}{321} \quad \ldots(2)$$

$$\frac{18^{\text{th}} \text{ term of first A.P.}}{18^{\text{th}} \text{ term of second A.P.}} = \frac{a_1+17d_1}{a_2+17d_2} \quad \ldots(3)$$

From (2) and (3), we obtain

$$\frac{18^{\text{th}} \text{ term of first A.P.}}{18^{\text{th}} \text{ term of second A.P.}} = \frac{179}{321}$$

Thus, the ratio of 18th term of both the A.P.s is 179: 321.

P.T.O

23)

Answer.

Let $r^{\frac{a}{r}, a, ar}$ be the first three terms of the G.P.

$\frac{a}{r} + a + ar = \frac{39}{10} \quad \ldots(1)$

$\left(\frac{a}{r}\right)(a)(ar) = 1 \quad \ldots(2)$

From (2), we obtain

$a^3 = 1$

$\Rightarrow a = 1$ (Considering real roots only)

Substituting a = 1 in equation (1), we obtain

$\frac{1}{r} + 1 + r = \frac{39}{10}$

$\Rightarrow 1 + r + r^2 = \frac{39}{10}r$

$\Rightarrow 10 + 10r + 10r^2 - 39r = 0$

$\Rightarrow 10r^2 - 29r + 10 = 0$

$\Rightarrow 10r^2 - 25r - 4r + 10 = 0$

$\Rightarrow 5r(2r - 5) - 2(2r - 5) = 0$

$\Rightarrow (5r - 2)(2r - 5) = 0$

$\Rightarrow r = \frac{2}{5}$ or $\frac{5}{2}$

Thus, the three terms of G.P. are $\frac{5}{2}, 1,$ and $\frac{2}{5}$

24)

(a) $18n - 10$ (or equivalent)

(b) $\sum_{1}^{n}(18r - 10)$ (or equivalent)

(c) by use of GDC or algebraic summation or sum of an AP

$\sum_{1}^{15}(18r - 10) = 2010$

P.T.O

25)

(a) $S_n = \frac{n}{2}[2a + (n-1)d]$

$212 = \frac{16}{2}(2a + 15d) \quad (= 16a + 120d)$

n^{th} term is $a + (n-1)d$

$8 = a + 4d$

solving simultaneously:

$d = 1.5,\ a = 2$

(b) $\frac{n}{2}[4 + 1.5(n-1)] > 600$

$\Rightarrow 3n^2 + 5n - 2400 > 0$

$\Rightarrow n > 27.4\ldots,\ (n < -29.1\ldots)$

Note: Do not penalize improper use of inequalities.

$\Rightarrow n = 28$

26)

(a) 150000×1.035^{20} *(M1)(A1)*

$= \$298468$ *A1*

Note: Only accept answers to the nearest dollar. Accept \$298469.

[3 marks]

(b) attempt to look for a pattern by considering 1 year, 2 years *etc* *(M1)*

recognising a geometric series with first term P and common ratio 1.02 *(M1)*

EITHER

$P + 1.02P + \ldots + 1.02^{19}P \left(= P\left(1 + 1.02 + \ldots + 1.02^{19}\right)\right)$ *A1*

OR

explicitly identify $u_1 = P$, $r = 1.02$ and $n = 20$ (may be seen as S_{20}). *A1*

THEN

$S_{20} = \frac{\left(1.02^{20} - 1\right)P}{(1.02 - 1)}$ *AG*

[3 marks]

(c) $24.297\ldots P = 298468$ *(M1)(A1)*

$P = 12284$ *A1*

27)

(a) correct substitution into infinite sum *(A1)*

eg $200 = \frac{4}{1-r}$

$r = 0.98$ (exact) **A1** **N2**

[2 marks]

(b) correct substitution *(A1)*

$\frac{4(1-0.98^8)}{1-0.98}$

29.8473

29.8 **A1** **N2**

[2 marks]

(c) attempt to set up inequality (accept equation) *(M1)*

eg $\frac{4(1-0.98^n)}{1-0.98} > 163$, $\frac{4(1-0.98^n)}{1-0.98} = 163$

correct inequality for n (accept equation) or crossover values *(A1)*

eg $n > 83.5234$, $n = 83.5234$, $S_{83} = 162.606$ **and** $S_{84} = 163.354$

$n = 84$ **A1** **N1**

[3 marks]

28)

(a) valid approach *(M1)*

eg $11-5$, $11 = 5 + d$

$d = 6$ **A1** **N2**

[2 marks]

(b) valid approach *(M1)*

eg $u_2 - d$, $5-6$, $u_1 + (3-1)(6) = 11$

$u_1 = -1$ **A1** **N2**

[2 marks]

(c) correct substitution into sum formula

eg $\frac{20}{2}\left(2(-1)+19(6)\right)$, $\frac{20}{2}(-1+113)$ *(A1)*

$S_{20} = 1120$ **A1** **N2**

[2 marks]

P.T.O

29)

(a) $\dfrac{100}{1-r}=2000$	M1		Correct formula and attempt to solve
$r=19/20$	A1		
$ar=95$	A1√	[3]	For $100\times r$
(b) **(i)** $a+2d=90,\ a+4d=80$			
$d=-5,\ a=100$	B1B1	[2]	
(ii) $a+md=0$	M1		Or use correct sum formula
$m=20$	A1	[2]	$m=20$ with no working scores 2
(iii) $\dfrac{n}{2}[200+(n-1)(-5)]=0$	M1		
$n=41$	A1	[2]	$n=41$ with no working scores 2 Do not penalise $n=0$

30)

	$a+(n-1)3=94$	**B1**	
	$\frac{n}{2}\left[2a+(n-1)3\right]=1420$ OR $\frac{n}{2}\left[a+94\right]=1420$	**B1**	
	Attempt elimination of a or n	**M1**	
	$3n^2-191n+2840\ (=0)$ OR $a^2-3a-598\ (=0)$	**A1**	3-term quadratic (not necessarily all on the same side)
	$n=40$ (only)	**A1**	
	$a=-23$ (only)	**A1**	Award 5/6 if a 2nd pair of solutions (71/3, 26) is given in addition or if given as the only answer.
		6	