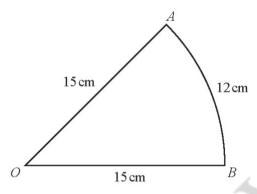
Additional Mathematics- 0606 Circular Measure Questions

1)



The diagram shows a sector AOB of a circle, centre O, radius 15 cm. The length of the arc AB is 12 cm.

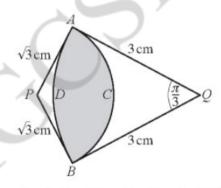
(i) Find, in radians, angle AOB.

[2]

(ii) Find the area of the sector AOB.

[2]

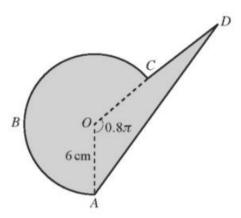
2)



In the diagram, ACB is an arc of a circle with centre P, and ADB is an arc of a circle with centre Q.

Angle $AQB = \frac{\pi}{3}$, AQ = BQ = 3 cm and $AP = BP = \sqrt{3}$ cm.

(i) Show that angle
$$APB = \frac{2\pi}{3}$$
. [2]

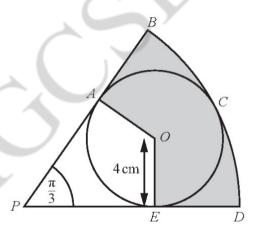


The diagram represents a company logo ABCDA, consisting of a sector OABCO of a circle, centre O and radius 6 cm, and a triangle AOD. Angle $AOC = 0.8\pi$ radians and C is the mid-point of OD. Find

(i) the perimeter of the logo, [7]

(ii) the area of the logo. [5]

4)



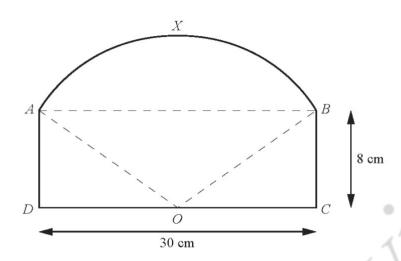
The diagram shows a circle, centre O, radius 4 cm, enclosed within a sector PBCDP of a circle, centre P. The circle centre O touches the sector at points A, C and E. Angle BPD is $\frac{\pi}{3}$ radians.

(i) Show that
$$PA = 4\sqrt{3}$$
 cm and $PB = 12$ cm. [2]

Find, to 1 decimal place,

(ii) the area of the shaded region, [4]

(iii) the perimeter of the shaded region. [4]



The diagram shows a rectangle ABCD and an arc AXB of a circle with centre at O, the mid-point of DC. The lengths of DC and BC are 30 cm and 8 cm respectively. Find

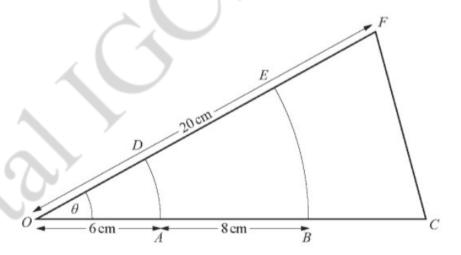
(i) the length of OA, [2]

(ii) the angle AOB, in radians, [2]

(iii) the perimeter of figure ADOCBXA, [2]

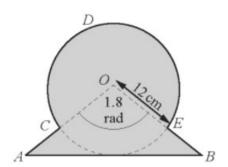
(iv) the area of figure ADOCBXA. [2]

6)



In the diagram AD and BE are arcs of concentric circles centre O, where OA = 6 cm and AB = 8 cm. The area of the region ABED is 32 cm^2 . The triangle OCF is isosceles with OC = OF = 20 cm.

- (i) Find the angle θ in radians. [3]
- (ii) Find the perimeter of the region BCFE. [5]



The diagram shows an isosceles triangle AOB and a sector OCDEO of a circle with centre O. The line AB is a tangent to the circle. Angle AOB = 1.8 radians and the radius of the circle is $12 \, \text{cm}$.

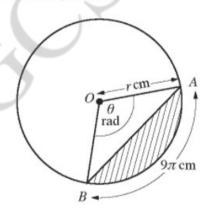
(i) Show that the distance AC = 7.3 cm to 1 decimal place. [2]

(ii) Find the perimeter of the shaded region. [6]

(iii) Find the area of the shaded region. [4]

8)

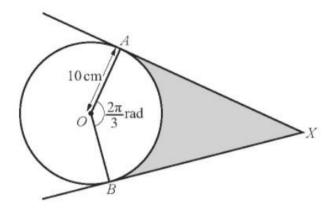
The figure shows a circle, centre O, radius r cm. The length of the arc AB of the circle is 9π cm. Angle AOB is θ radians and is 3 times angle OBA.



(i) Show that $\theta = \frac{3\pi}{5}$. [2]

(ii) Find the value of r. [2]

(iii) Find the area of the shaded region. [3]

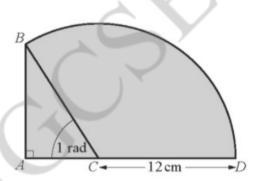


The figure shows a circle, centre O, with radius 10 cm. The lines XA and XB are tangents to the circle at A and B respectively, and angle AOB is $\frac{2\pi}{3}$ radians.

(i) Find the perimeter of the shaded region. [3]

(ii) Find the area of the shaded region. [4]

10)

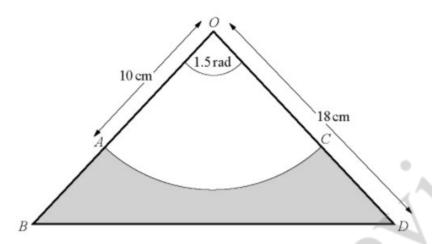


The diagram shows a right-angled triangle ABC and a sector CBDC of a circle with centre C and radius $12 \, \text{cm}$. Angle ACB = 1 radian and ACD is a straight line.

(i) Show that the length of AB is approximately 10.1 cm. [1]

(ii) Find the perimeter of the shaded region. [5]

(iii) Find the area of the shaded region. [4]

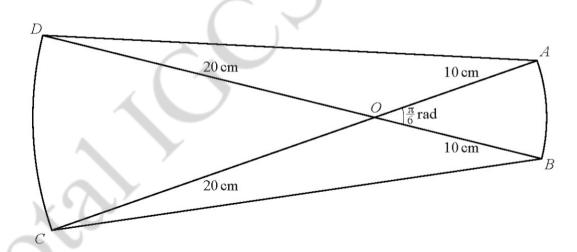


The diagram shows an isosceles triangle OBD in which OB = OD = 18 cm and angle BOD = 1.5 radians. An arc of the circle, centre O and radius 10 cm, meets OB at A and OD at C.

(i) Find the area of the shaded region. [3]

(ii) Find the perimeter of the shaded region. [4]

12)



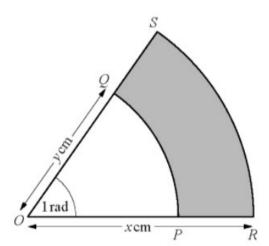
The diagram shows four straight lines, AD, BC, AC and BD. Lines AC and BD intersect at O such that angle AOB is $\frac{\pi}{6}$ radians. AB is an arc of the circle, centre O and radius 10 cm, and CD is an arc of the circle, centre O and radius 20 cm.

(i) Find the perimeter of ABCD.

[4]

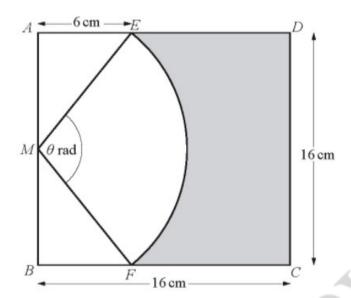
P.T.O

13)



In the diagram PQ and RS are arcs of concentric circles with centre O and angle POQ = 1 radian. The radius of the larger circle is x cm and the radius of the smaller circle is y cm.

- (i) Given that the perimeter of the shaded region is $20 \,\mathrm{cm}$, express y in terms of x. [2]
- (ii) Given that the area of the shaded region is $16 \,\mathrm{cm}^2$, express y^2 in terms of x^2 . [2]
- (iii) Find the value of x and of y. [4]

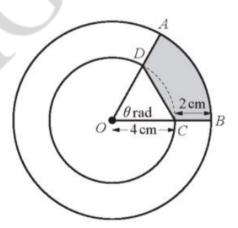


The diagram shows a square ABCD of side 16 cm. M is the mid-point of AB. The points E and F are on AD and BC respectively such that AE = BF = 6 cm. EF is an arc of the circle centre M, such that angle EMF is θ radians.

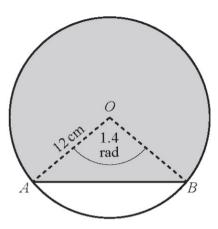
- (i) Show that $\theta = 1.855$ radians, correct to 3 decimal places. [2]
- (ii) Calculate the perimeter of the shaded region. [4]
- (iii) Calculate the area of the shaded region. [3]

15)

The diagram shows two concentric circles, centre O, radii 4 cm and 6 cm. The points A and B lie on the larger circle and the points C and D lie on the smaller circle such that ODA and OCB are straight lines.



- (i) Given that the area of triangle OCD is $7.5 \, \mathrm{cm}^2$, show that $\theta = 1.215$ radians, to 3 decimal places. [2]
- (ii) Find the perimeter of the shaded region. [4]
- (iii) Find the area of the shaded region. [3]



The diagram shows a circle with centre O and a chord AB. The radius of the circle is 12 cm and angle AOB is 1.4 radians.

(i) Find the perimeter of the shaded region.

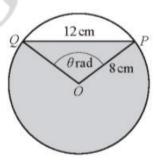
[5]

[2]

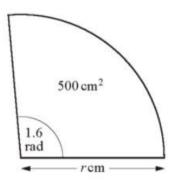
P.T.O

17)

The diagram shows a circle, centre O, radius 8 cm. Points P and Q lie on the circle such that the chord PQ = 12 cm and angle $POQ = \theta$ radians.



- (i) Show that $\theta = 1.696$, correct to 3 decimal places.
- (ii) Find the perimeter of the shaded region. [3]
- (iii) Find the area of the shaded region. [3]



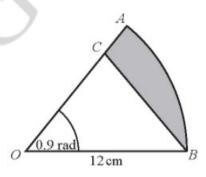
The diagram shows a sector of a circle of radius r cm. The angle of the sector is 1.6 radians and the area of the sector is $500 \, \text{cm}^2$.

(i) Find the value of r. [2]

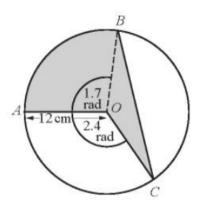
(ii) Hence find the perimeter of the sector. [2]

P.T.O 19)

The diagram shows a sector, AOB, of a circle centre O, radius 12 cm. Angle AOB = 0.9 radians. The point C lies on OA such that OC = CB.



- (i) Show that OC = 9.65 cm correct to 3 significant figures. [2]
- (ii) Find the perimeter of the shaded region. [3]
- (iii) Find the area of the shaded region. [3]



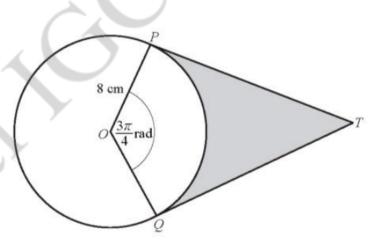
The diagram shows a circle, centre O, radius 12 cm. The points A, B and C lie on the circumference of this circle such that angle AOB is 1.7 radians and angle AOC is 2.4 radians.

(i) Find the area of the shaded region. [4]

(ii) Find the perimeter of the shaded region. [5]

P.T.O

21)

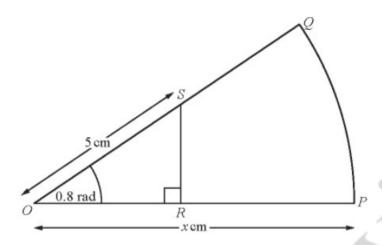


The diagram shows a circle, centre O, radius 8 cm. The points P and Q lie on the circle. The lines PT and QT are tangents to the circle and angle $POQ = \frac{3\pi}{4}$ radians.

(i) Find the length of PT. [2]

(ii) Find the area of the shaded region. [3]

(iii) Find the perimeter of the shaded region. [2]



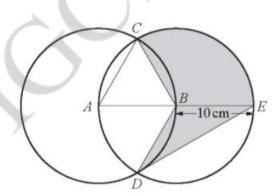
The diagram shows a sector OPQ of a circle with centre O and radius x cm. Angle POQ is 0.8 radians. The point S lies on OQ such that OS = 5 cm. The point S lies on S such that angle S is a right angle. Given that the area of triangle S is one-fifth of the area of sector S ind

(i) the area of sector OPQ in terms of x and hence show that the value of x is 8.837 correct to 4 significant figures, [5]

(ii) the perimeter of PQSR, [3]

(iii) the area of PQSR. [2]

23)



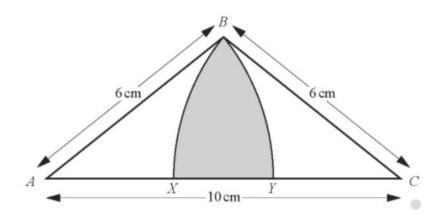
The diagram shows two circles, centres A and B, each of radius $10 \, \text{cm}$. The point B lies on the circumference of the circle with centre A. The two circles intersect at the points C and D. The point E lies on the circumference of the circle centre B such that ABE is a diameter.

(i) Explain why triangle ABC is equilateral. [1]

(ii) Write down, in terms of π , angle CBE. [1]

(iii) Find the perimeter of the shaded region. [5]

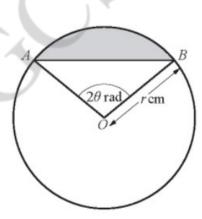
(iv) Find the area of the shaded region. [3]



The diagram shows an isosceles triangle ABC such that AC = 10 cm and AB = BC = 6 cm. BX is an arc of a circle, centre C, and BY is an arc of a circle, centre A.

- (i) Show that angle ABC = 1.970 radians, correct to 3 decimal places. [2]
- (ii) Find the perimeter of the shaded region. [4]
- (iii) Find the area of the shaded region. [3]

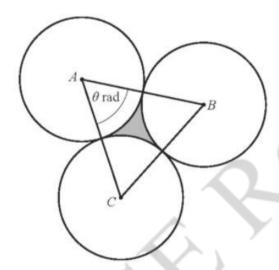
P.T.O 25)



The diagram shows a circle, centre O, radius r cm. The points A and B lie on the circle such that angle $AOB = 2\theta$ radians.

(i) Find, in terms of r and θ , an expression for the length of the chord AB. [1]

- (ii) Given that the perimeter of the shaded region is 20 cm, show that $r = \frac{10}{\theta + \sin \theta}$. [2]
- (iii) Given that r and θ can vary, find the value of $\frac{dr}{d\theta}$ when $\theta = \frac{\pi}{6}$. [4]
- (iv) Given that r is increasing at the rate of $15\,\mathrm{cm\,s^{-1}}$, find the corresponding rate of change of θ when $\theta = \frac{\pi}{6}$.

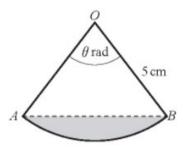


The diagram shows 3 circles with centres A, B and C, each of radius 5 cm. Each circle touches the other two circles. Angle BAC is θ radians.

(i) Write down the value of θ . [1]

(ii) Find the area of the shaded region between the circles. [4]

27)



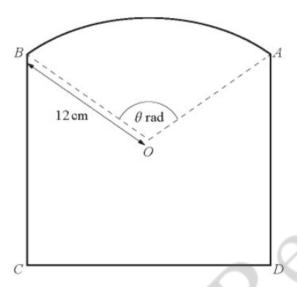
The diagram shows a sector of a circle with centre O and radius 5 cm. The length of the arc AB is 7 cm. Angle AOB is θ radians.

(i) Explain why θ must be greater than 1 radian. [1]

(ii) Find the value of θ . [2]

(iii) Calculate the area of the sector AOB. [2]

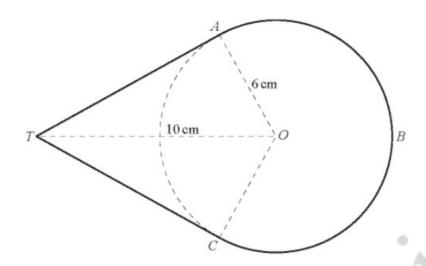
(iv) Calculate the area of the shaded segment. [2]



The diagram shows a sector AOB of the circle, centre O, radius $12\,\mathrm{cm}$, together with points C and D such that ABCD is a rectangle. The angle AOB is θ radians and the perimeter of the sector AOB is $47\,\mathrm{cm}$.

- (i) Show that $\theta = 1.92$ radians correct to 2 decimal places. [2]
- (ii) Find the length of CD. [2]
- (iii) Given that the total area of the shape is 425 cm², find the length of AD. [5]

29)



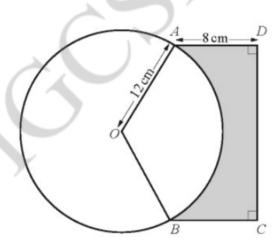
The points A, B and C lie on a circle centre O, radius 6 cm. The tangents to the circle at A and C meet at the point T. The length of OT is 10 cm. Find

(i) the angle TOA in radians, [2]

(ii) the area of the region TABCT, [6]

(iii) the perimeter of the region TABCT. [2]

30)

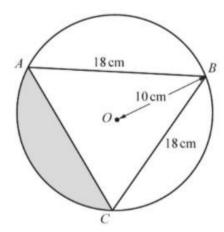


The diagram shows a circle, centre O, radius 12 cm. The points A and B lie on the circumference of the circle and form a rectangle with the points C and D. The length of AD is 8 cm and the area of the minor sector AOB is $150 \, \text{cm}^2$.

(i) Show that angle AOB is 2.08 radians, correct to 2 decimal places. [2]

(ii) Find the area of the shaded region ADCB. [6]

(iii) Find the perimeter of the shaded region ADCB. [3]



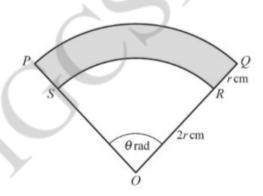
The diagram shows a circle centre O, radius 10 cm. The points A, B and C lie on the circumference of the circle such that AB = BC = 18 cm.

(i) Show that angle AOB = 2.24 radians correct to 2 decimal places. [3]

(ii) Find the perimeter of the shaded region. [5]

(iii) Find the area of the shaded region. [3]

32)



The diagram shows a sector OPQ of the circle centre O, radius 3r cm. The points S and R lie on OP and OQ respectively such that ORS is a sector of the circle centre O, radius 2r cm. The angle $POQ = \theta$ radians. The perimeter of the shaded region PQRS is 100 cm.

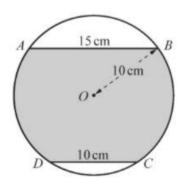
(i) Find θ in terms of r.

(ii) Hence show that the area, $A \text{ cm}^2$, of the shaded region *PQRS* is given by $A = 50r - r^2$. [2]

(iii) Given that r can vary and that A has a maximum value, find this value of A. [2]

(iv) Given that A is increasing at the rate of 3 cm² s⁻¹ when r = 10, find the corresponding rate of change of r.
[3]

(v) Find the corresponding rate of change of θ when r = 10. [3]



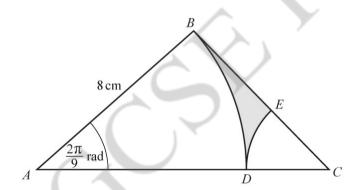
The diagram shows a circle with centre O and radius 10 cm. The points A, B, C and D lie on the circle such that the chord AB = 15 cm and the chord CD = 10 cm. The chord AB is parallel to the chord DC.

(i) Show that the angle AOB is 1.70 radians correct to 2 decimal places. [2]

(ii) Find the perimeter of the shaded region. [4]

(iii) Find the area of the shaded region. [4]

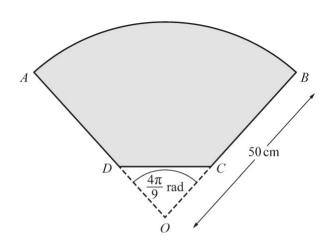
34)



The diagram shows a right-angled triangle ABC with AB=8 cm and angle $ABC=\frac{\pi}{2}$ radians. The points D and E lie on AC and BC respectively. BAD and ECD are sectors of the circles with centres A and C respectively. Angle $BAD=\frac{2\pi}{9}$ radians.

(i) Find the area of the shaded region. [6]

(ii) Find the perimeter of the shaded region. [3]

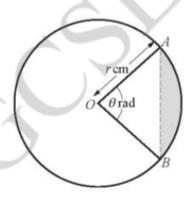


The diagram shows a company logo, ABCD. The logo is part of a sector, AOB, of a circle, centre O and radius 50 cm. The points C and D lie on OB and OA respectively. The lengths AD and BC are equal and AD:AO is 7:10. The angle AOB is $\frac{4\pi}{9}$ radians.

(i) Find the perimeter of *ABCD*. [5]

(ii) Find the area of ABCD. [3]

36)

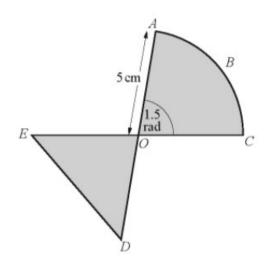


The diagram shows a circle with centre O and radius rcm. The minor are AB is such that angle AOB is θ radians. The area of the minor sector AOB is $48 \, \text{cm}^2$.

(i) Show that
$$\theta = \frac{96}{r^2}$$
. [2]

(ii) Given that the minor arc AB has length 12 cm, find the value of r and of θ . [3]

(iii) Using your values of r and θ , find the area of the shaded region. [2]

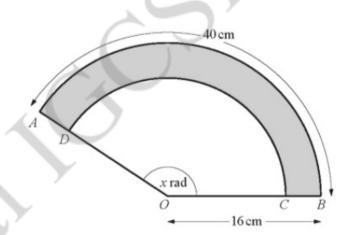


In the diagram, ABC is an arc of the circle centre O, radius 5 cm, and angle AOC is 1.5 radians. AD and CE are diameters of the circle and DE is a straight line.

(i) Find the total perimeter of the shaded regions. [3]

(ii) Find the total area of the shaded regions. [3]

38)

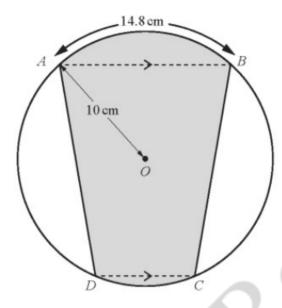


In the diagram AOB and DOC are sectors of a circle centre O. The angle AOB is x radians. The length of the arc AB is 40 cm and the radius OB is 16 cm.

(i) Find the value of x. [2]

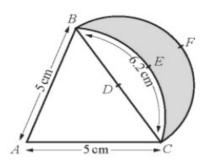
(ii) Find the area of sector AOB. [2]

(iii) Given that the area of the shaded region ABCD is 140 cm², find the length of OC. [3]



The diagram shows a circle, centre O, radius $10 \, \mathrm{cm}$. The points A, B, C and D lie on the circumference of the circle such that AB is parallel to DC. The length of the minor are AB is $14.8 \, \mathrm{cm}$. The area of the minor sector ODC is $21.8 \, \mathrm{cm}^2$.

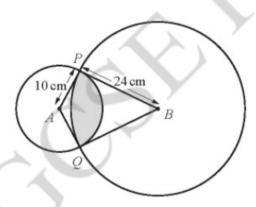
- (i) Write down, in radians, angle AOB. [1]
- (ii) Find, in radians, angle DOC. [2]
- (iii) Find the perimeter of the shaded region. [4]
- (iv) Find the area of the shaded region. [3]



The diagram shows an isosceles triangle ABC, where AB = AC = 5 cm. The arc BEC is part of the circle centre A and has length 6.2 cm. The point D is the midpoint of the line BC. The arc BFC is a semi-circle centre D.

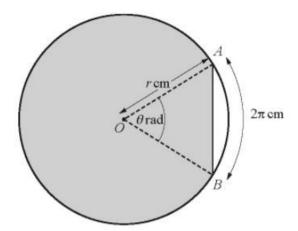
- (i) Show that angle BAC is 1.24 radians. [1]
- (ii) Find the perimeter of the shaded region. [3]
- (iii) Find the area of the shaded region. [4]

41)



The diagram shows a circle, centre A, radius $10 \, \mathrm{cm}$, intersecting a circle, centre B, radius $24 \, \mathrm{cm}$. The two circles intersect at the points P and Q. The radii AP and AQ are tangents to the circle with centre B. The radii BP and BQ are tangents to the circle with centre A.

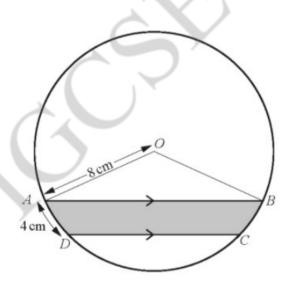
- (i) Show that angle PAQ is 2.35 radians, correct to 3 significant figures. [2]
- ii) Find angle PBQ in radians. [1]
- (iii) Find the perimeter of the shaded region. [3]
- (iv) Find the area of the shaded region. [4]



The diagram shows a circle, centre O of radius r cm, and a chord AB. Angle $AOB = \theta$ radians. The length of the major are AB is 5 times the length of the minor are AB. The minor are AB has length 2π cm.

- (i) Find the value of θ and of r.
- (ii) Calculate the exact perimeter of the shaded segment. [2]
- (iii) Calculate the exact area of the shaded segment. [4]

43)



The diagram shows a circle, centre O, radius 8 cm. The points A, B, C and D lie on the circumference of the circle such that AB is parallel to DC. The length of the arc AD is 4 cm and the length of the chord AB is 15 cm.

- (i) Find, in radians, angle AOD. [1]
- (ii) Hence show that angle DOC = 1.43 radians, correct to 2 decimal places. [3]
- (iii) Find the perimeter of the shaded region. [3]
- (iv) Find the area of the shaded region. [4]

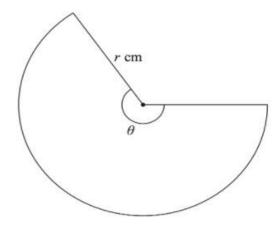


Diagram NOT accurately drawn

Figure 1

Figure 1 shows a sector of a circle. The circle has radius r cm and the sector has angle θ radians. The sector has an arc length of 18π cm and an area of 126π cm².

Find

- (i) the value of r,
- (ii) the exact value of θ .

(5)

45) – extra question (test your concept)

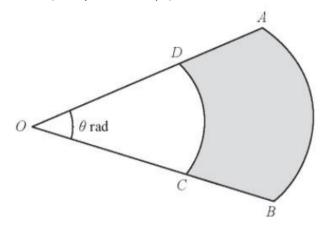


Diagram NOT accurately drawn

Figure 1

Figure 1 shows a sector OAB of the circle with centre O and radius 10 cm.

The points C and D lie on OB and OA respectively and CD is an arc of the circle with centre O and radius 6 cm. The size of angle AOB is θ radians. The shaded region is bounded by the arcs AB and CD and the lines AD and BC.

The area of the shaded region is S cm2.

(a) Show that $S = 32\theta$.

(3)

The size of angle AOB is increasing at a constant rate of 0.2 rad/s.

(b) Find the rate of increase of S.

(2)

When the area of the shaded region is 20 cm2

(c) calculate the perimeter of the shaded region.

(5)