Additional Mathematics- 0606
Circular Measure Mark scheme

1)

(i) $12 = 15\theta$, $\theta = 0.8$ rads

(ii) Area = $\frac{1}{2}15^2(0.8)$

leading to 90 (cm^2)

2)

(i) AB = 3 or $\frac{\sin \angle APQ}{3} = \frac{\sin \pi/6}{\sqrt{3}}$

Correct use of trigonometry to APB = $\frac{2\pi}{3}$

(ii) Uses $s = r\theta$

3.14 (π) or 3.63 $\left(\frac{2\sqrt{3}\pi}{3}\right)$

6.77 $\left(\pi + \frac{2\sqrt{3}\pi}{3}\right)$

(iii) Uses $\frac{1}{2}r^2\theta$ or $\frac{1}{2}rs$

Uses $\frac{1}{2}r^2 \sin\theta$ or area kite

Either 4.71 (1.5π) and 3.14 (π),

or 3.90 $\left(\frac{9\sqrt{3}}{4}\right)$ and 1.30 $\left(\frac{3\sqrt{3}}{4}\right)$ **or** 5.20 $(3\sqrt{3})$

Complete plan

2.65 to 2.66 $(2.5\pi - 3\sqrt{3})$

P.T.O

3)

(i) Sector angle $= 1.2\pi$

$OD = 12$

$AD^2 = 12^2 + 6^2 - 2\times 12\times 6\cos 0.8\pi$

$AD = 17.2$

Uses $s = 6\times(1.2\pi) = (7.2\pi)$ (or 22.6)

Complete plan $(AD + r\theta + 6)$ or $(17.2 + 7.2\pi + 6)$

45.8

(ii) $\Delta\, AOD = \frac{1}{2}\times 6\times 12\sin 0.8\pi$

21.2

Uses $A = \frac{1}{2}\times 6^2\times(1.2\pi)$

21.6π or 67.8 or 67.9

89.0 or 89

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4)

(i) $\tan\frac{\pi}{6} = \frac{4}{PA}$, $PA = 4\sqrt{3}$

$PB = \dfrac{4}{\sin\frac{\pi}{6}} + 4$, $PB = 12$

allow equivalent methods

(ii) Sector area $= \frac{1}{2}12^2\times\frac{\pi}{3}$

Area of kite $= 2\times\frac{1}{2}\times 4\sqrt{3}\times 4$

Shaded area = 47.7

(iii) $P = \left(12\times\frac{\pi}{3}\right) + 2\left(12 - 4\sqrt{3}\right) + 2(4)$

$= 30.7$

P.T.O

5)

(i) $\sqrt{8^2 + 15^2}$
$AO = 17$

(ii) $AOB = \pi - 2\arctan(8/15)$ or $\arccos\left(\frac{17^2 + 17^2 - 30^2}{2 \times 17 \times 17}\right)$

$AOB = 2.16$

(iii) Complete, correct plan with $s = r\theta$
82.7

(iv) Complete, correct plan with $A = \frac{1}{2}r^2\theta$

432

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6)

(i) $\frac{1}{2}(14^2 - 6^2)\theta = 32$
0.4

(ii) $\frac{CF}{2} = 20 \sin 0.2$ or $CF^2 = 20^2 + 20^2 - 2 \times 20 \times 20 \cos 0.4$
(CF=)7.95
Uses $s = r\theta$
Complete plan including $s = r\theta$
25.5 or 25.6

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7)

(i) $OA = \dfrac{12}{\cos 0.9}$

$AC = 19.3 - 12 = 7.3$

(ii) Complete method for major arc $(2\pi - 1.8) \times 12$

53.8

$AB = 2 \times 12\tan 0.9$ or cosine rule

30.2

Complete plan $(53.8 + 30.2 + 2 \times 7.3)$

98.6

(iii) Complete method for major sector $\frac{1}{2} \times 12^2 \times (2\pi - 1.8)$

$\frac{1}{2} \times 19.3^2 \times \sin 1.8$ or $\frac{1}{2} \times 30.2 \times 12$

323 or 181

504

8)

(i) $\Delta OBA : \theta + 2\left(\dfrac{\theta}{3}\right) = \pi$

(ii) $9\pi = r \times \dfrac{3\pi}{5}$

$r = 15$

(iii) Area $= \left(\dfrac{1}{2} \times 15^2 \times \dfrac{3\pi}{5}\right) - \left(\dfrac{1}{2} \times 15^2 \times \sin \dfrac{3\pi}{5}\right)$

$= 105$

9)

(i) arc $AB = \frac{20\pi}{3}$ or 20.94, 20.9

$\tan\frac{\pi}{3} = \frac{AX}{10}$, AX = $10\sqrt{3}$, 17.3 (or XB)

Perimeter = awrt 55.6 or $20\sqrt{3} + \frac{20\pi}{3}$

(ii) Area of sector AOB $= \frac{1}{2}10^2 \frac{2\pi}{3}$ or 104.7

or 105

Area of $OAXB$ = $100\sqrt{3}$ or 173.2

Shaded area = awrt 68.5 or $100\sqrt{3} - \frac{100\pi}{3}$

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10)

(i) $AB = 12\sin 1 = 10.1$ **AG**	**B1**
(ii) $AC = 12\cos 1 = 6.48$ or 6.5 oe	**B1**
$\angle BCD = 2.14$ or $(\pi - 1)$	**B1**
Use $s = r\theta$ (25.7)	**M1**
Use complete plan	**M1**
54.3	**A1**
(iii) Area $ACB = \frac{1}{2} \times$ base $\times$ height	**M1**
Area $BCD = \frac{1}{2}r^2\theta$	**M1**
154 or 32.7 (or 33)	**A1**
187	**A1**
	[10]

11)

(i) Area = $\frac{1}{2}$ 18^2 sin 1.5 – $\frac{1}{2}$ $10^2(1.5)$

= 161.594 – 75

= 86.6

(or area of triangle = $\frac{1}{2}$ × 24.539 × 13.170)

(ii) AC = 15 or 10 × 1.5
LBD = 36 sin 0.75

$BD = \sqrt{18^2 + 18^2 - (2\times18\times18\cos 1.5)}$
= 24.5

Perimeter = 15 + 24.5 + 16
= 55.5

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12)

(i) $AD^2 = 20^2 + 10^2 - 2(20)(10)\cos\frac{5\pi}{6}$

Perimeter = $\frac{10\pi}{6} + \frac{20\pi}{6} + 2(29.1)$
= 73.9

(ii) Area =
$\frac{1}{2}10^2\left(\frac{\pi}{6}\right) + \frac{1}{2}20^2\left(\frac{\pi}{6}\right) + 2\left(\frac{1}{2}(10)(20)\sin\frac{5\pi}{6}\right)$

= 231

P.T.O **13)**

(i) Uses $s = r\theta$
$y = 3x - 20$

(ii) Uses $A = \frac{1}{2}r^2\theta$
$y^2 = x^2 - 32$

(iii) Eliminate y or x
$x^2 - 15x + 54 = 0$ or $y^2 - 5y - 14 = 0$
Solve 3 term quadratic
$x = 9$ and $y = 7$

14)

(i) **Either** $\tan\frac{\theta}{2} = \frac{8}{6}$

$\frac{\theta}{2} = 0.927...$

$\theta = 1.855$

Or Area of triangle $MEF = 48$

$\frac{1}{2} \times 10^2 \times \sin\theta = 48$

$\theta = 1.287, \pi - 1.287$

$\theta = 1.855$

Or $16^2 = 10^2 + 10^2 - (2 \times 10 \times \cos\theta)$

$\theta = 1.855$

(ii) radius = 10

$P = (10 \times 1.855) + 10 + 10 + 16$

$= 54.6$ or 54.5 or 54.55

(iii) $A = 256 - 2\left(\frac{1}{2} \times 8 \times 6\right) - \frac{1}{2}10^2(1.855)$

$= 115.25$ or 115.3 or 115

awrt 115

15)

(i) $\frac{1}{2}(4^2)\sin\theta = 7.5$

$\sin\theta = \frac{15}{16}$, $\theta = 1.215\ldots$

(ii) $\sin\frac{\theta}{2} = \frac{\frac{1}{2}CD}{4}$, $(CD = 4.567)$

Arc length = 6(1.215)

Perimeter = 2 + 2 + 6(1.215) + their *CD*

= awrt 15.9

(iii) Area = $\frac{1}{2}6^2(1.215) - 7.5$

= 14.4 (awrt)

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16)

(i) $AB^2 = 12^2 + 12^2 - 2\times12\times12\times\cos1.4$

15.4 to 15.5

$\theta = 2\pi - 1.4(= 4.88)$

Use $s = r\theta(= 58.6)$

74.1

(ii) (Sector) $\frac{1}{2}\times12^2\times(2\pi - 1.4)(= 352)$ or

$\pi\times12^2 - \frac{1}{2}\times12^2\times1.4$

(Triangle) $= \frac{1}{2}\times12\times12\times\sin1.4(= 70.9 \text{ or } 71)$

Area of **major** sector + Area of triangle

422 or 423

P.T.O **17)**

(i)	$\sin\frac{\theta}{2}=\frac{6}{8}$, $\frac{\theta}{2}=0.8481$ or better
	or $12^2=8^2+8^2-128\cos\theta$
	$\theta=1.6961$ or better
	or using areas $\frac{1}{2}\times 12\times 2\sqrt{7}=\frac{1}{2}8^2\sin\theta$ oe
	$\sin\theta=0.9922$, $\theta=1.4455$ or 1.6961
(ii)	Arc length = $(2\pi-1.696)\times 8$
	$(36.697 \text{ or } 36.7)$
	Perimeter = $12+(2\pi-1.696)\times 8$ $=48.7$
(iii)	Area $=\frac{8^2}{2}(2\pi-1.696)+\frac{8^2}{2}\sin 1.696$
	$=178.5,\ 178.6$, awrt 179
	Alternative: Area = $\pi 8^2-\left(\frac{1}{2}8^2(1.696)-\frac{8^2}{2}\sin 1.696\right)$

18)

(i)	$500=\frac{1}{2}r^2(1.6)$
	25 only
(ii)	*their* 25 + *their* 25 + *their* 25 × 1.6 or better
	90

19)

(i)	$\cos 0.9 = \dfrac{6}{OC}$ or $\dfrac{OC}{\sin 0.9} = \dfrac{12}{\sin(\pi - 1.8)}$ $OC = \dfrac{6}{\cos 0.9} = 9.652...$ or $OC = \dfrac{12\sin 0.9}{\sin(\pi - 1.8)} = 9.652...$
(ii)	Perimeter = $(0.9 \times 12) + 9.652 + (12 - 9.652)$ $= 22.8$
(iii)	Area $= \left(\dfrac{1}{2} \times 12^2 \times 0.9\right) - \left(\dfrac{1}{2} \times 9.652^2 \sin(\pi - 1.8)\right)$ 64.8 – 45.36 = 19.4 to 19.5 Alternative Method: $\dfrac{1}{2}(12 - 9.652) \times 9.652 \times \sin 1.8$ $\dfrac{1}{2}12^2(0.9 - \sin 0.9)$ 11.04 + 8.40 Area =19.4 to 19.5

<u>P.T.O</u> <u>20)</u>

(i)	Area = $\left(\frac{1}{2}\times 12^2 \times 1.7\right)+\left(\frac{1}{2}\times 12^2 \sin(2\pi - 1.7 - 2.4)\right)$ = awrt 181
(ii)	$BC^2 = 12^2 + 12^2 - (2\times 12\times 12\cos 2.1832)$ or $BC = 2\times 12\times \sin\left(\frac{2\pi - 4.1}{2}\right)$ $BC = 21.296$ Perimeter = $(12\times 1.7)+12+12+21.296$ = 65.7

21)

(i)	$\frac{PT}{8}=\tan\left(\frac{3\pi}{8}\right)$ oe $PT = 19.3$
(ii)	$\frac{1}{2}\times 8^2 \times \frac{3\pi}{4}$ oe (75.4) $8\tan\left(\frac{3\pi}{8}\right)\times 8 -$ *their* sector oe (=154.5-'75.4') 79.1
(iii)	$8\left(\frac{3\pi}{4}\right)$ oe (18.8) $\left[6\pi + 16\tan\left(\frac{3\pi}{8}\right)\right] = 57.5$

P.T.O 22)

(i)	Area of sector = $\frac{1}{2}\times x^2\times 0.8\left(=0.4x^2\text{ cm}^2\right)$ $SR=5\sin 0.8\,(=3.59)$ or $OR=5\cos 0.8\,(=3.48)$ Area of triangle = $\frac{1}{2}5\cos 0.8\times 5\sin 0.8=6.247\text{ cm}^2$ $0.08x^2=6.247$ $x=8.837\text{ cm}$ AG
(ii)	$SQ=8.84-5\,(=3.84\text{ cm})$ $PR=8.84-5\cos 0.8\,(=5.35\text{ or }5.36\text{ cm})$ $PQ=8.84\times 0.8\,(=7.07\text{ cm})$ Perimeter = 19.84 to 19.86 cm or rounded to 19.8 or 19.9
(iii)	Area $PQSR=4\times 6.247$ $=25\text{ cm}^2$

23)

(i)	All sides are equal to the radii of the circles which are also equal
(ii)	Angle $CBE=\frac{2\pi}{3}$
(iii)	$DE=10\sqrt{3}$ Arc $CE=10\times\frac{2\pi}{3}$ Perimeter $=20+10\sqrt{3}+\frac{20\pi}{3}$ $=58.3$ or 58.2
(iv)	Area of sector: $\frac{1}{2}\times 10^2\times\frac{2\pi}{3}=\frac{100\pi}{3}$ Area of triangle: $\frac{1}{2}\times 10^2\times\sin\frac{2\pi}{3}=25\sqrt{3}$ Area $=\frac{100\pi}{3}+25\sqrt{3}$ or awrt 148

24)

(i)	$10^2 = 6^2 + 6^2 - 2\times 6\times 6\times \cos ABC$ or $\sin\left(\frac{ABC}{2}\right) = \frac{5}{6}$ or $ABC = \pi - \sin^{-1}\frac{10\sqrt{11}}{36}$ $ABC = 1.9702$
(ii)	$XY = 2$ Arc length $6\left(\frac{\pi - 1.970}{2}\right)$ oe Perimeter = $2 + 2\left(6\left(\frac{\pi - 1.970}{2}\right)\right)$ $= 9.03$
(iii)	$\left(\frac{1}{2}\times 6^2\left(\frac{\pi - 1.970}{2}\right) - \frac{1}{2}\times 5\times \sqrt{11}\right)\times 2$ = 4.50 or 4.51 or better

P.T.O **25)**

(i)

$AB = 2r\sin\theta$

or $\sqrt{r^2 + r^2 - 2r^2\cos 2\theta}$

or $\dfrac{r\sin 2\theta}{\sin\left(\dfrac{\pi}{2} - \theta\right)}$

or $\dfrac{r\sin 2\theta}{\cos\theta}$

(ii)

$2r\sin\theta + 2r\theta = 20$

$r = \dfrac{10}{\theta + \sin\theta}$

(iii)

$\dfrac{dr}{d\theta} = -\dfrac{10(1+\cos\theta)}{(\theta+\sin\theta)^2}$

When $\theta = \dfrac{\pi}{6}$, $\dfrac{dr}{d\theta} = -17.8$

(iv)

$\dfrac{dr}{dt} = 15$

$\dfrac{d\theta}{dt} = \dfrac{dr}{dt} \div \dfrac{dr}{d\theta}$

$\dfrac{d\theta}{dt} = -0.842$

26)

(i)

$\dfrac{\pi}{3}$ isw

(ii)

[Area triangle ABC =] $\dfrac{1}{2} \times 10^2 \times \sin\left(their\dfrac{\pi}{3}\right)$ oe

[Area 1 sector =] $\dfrac{1}{2} \times 5^2 \times their\dfrac{\pi}{3}$ oe

or $\pi \times 5^2 \times \dfrac{their\,60°}{360}$

Complete correct plan

4.03(1...) or $25\sqrt{3} - \dfrac{25\pi}{2}$ isw

27)

(i)	Valid explanation
(ii)	$7 = 5\theta$ $\theta = 1.4$ oe
(iii)	$\frac{1}{2} \times 5^2 \times their 1.4$ oe 17.5oe
(iv)	[triangle area =] $\frac{1}{2} \times 5^2 \times \sin their 1.4$ or 12.3 to 12.32 or for [$\frac{1}{2}$ × base × height=] $\frac{1}{2} \times 6.4[4...] \times 3.8[2...]$ oe 5.18 to 5.2 inclusive

P.T.O **28)**

(i)

$47-24=12\theta$

$\theta=\frac{23}{12}$, so $\theta=1.917$ or better

$\theta=1.92$ to 2dp

(ii)

$\sin\frac{\theta}{2}=\frac{CD/2}{12}$

CD = awrt 19.6 or 19.7

(iii)

Area of sector = awrt 138
Area of triangle AOB = awrt 67 or 68
Area of segment = awrt 70 or 71

$AD\times AB$ + segment area = 425
leading to AD = awrt 18.1 or 18.0

Alternative method:
Area of sector = awrt 138
Difference in length between BC (or AD) and OM where M is the midpoint of CD = 6.88, allow awrt 6.9
Remaining area consists of two trapezia each of width 9.85 and each of area 143.4

$\frac{1}{2}(2BC-6.88)\times 9.85=143.4$ oe

leading to AD = awrt 18.1 or 18.0

29)

(i)

$\cos TOA=\frac{6}{10}\rightarrow$

$TOA=0.927$

(ii)

area of major sector =

$\frac{1}{2}6^2\left(2\pi-2\times their 0.927\right)$ (= 79.7)

area of half kite = $\frac{1}{2}(6)\sqrt{10^2-6^2}$ (=24)

area of kite $\times 2$ (=48)

complete correct plan
awrt 128

(iii)

arc length =

$6\times(2\pi-2\times their 0.927)+2\times\sqrt{10^2-6^2}$)

awrt 42.6

30)

(i)	$\frac{1}{2} \times 12^2 \times \theta = 150$
	$\theta = 2.083$, so $\theta = 2.08$ to 2dp
(ii)	Area of triangle AOB = $\frac{1}{2} \times 12^2 \sin 2.08$
	Area of segment = $150 - \frac{1}{2} \times 12^2 \times \sin 2.08$
	$\sin 1.04 = \frac{\frac{AB}{2}}{12}$
	AB = awrt 20.7
	Shaded area = *their* $AB \times 8$ – *their* segment area
	awrt 78.4 or 78.5
(iii)	Arc AB = 25 or 24.96
	Perimeter= 25 + *their* AB + 16
	awrt 61.7

P.T.O 31)

(i)	**Either** $18^2 = 10^2 + 10^2 - 200\cos AOB$	M1	Attempt to use cosine rule
	$\cos AOB = -0.62$	A1	Allow unsimplified
	$AOB = 2.2395$ or greater accuracy, so 2.24 (to 2 dp) or $AOB = 2.239...$ so 2.24 (to 2 dp) $AOB = 2.240$ so 2.24 (to 2 dp)	A1	Must justify 2 dp
(i)	**Or** $\sin\frac{AOB}{2} = \frac{9}{10}$ or $\tan\frac{AOB}{2} = \frac{9}{\sqrt{19}}$ or $\cos\frac{AOB}{2} = \frac{\sqrt{19}}{10}$	M1	Attempt at trig using a right angled triangle
	$\frac{AOB}{2} =$ awrt 1.12	A1	
	$AOB = 2.2395$ or greater accuracy, so 2.24 (to 2 dp) or $AOB = 2.239...$ so 2.24 (to 2 dp) $AOB = 2.240$ so 2.24 (to 2 dp)	A1	Must justify 2 dp
(ii)	$AOC = 2\pi - 2(2.2395)$ or $\frac{AOC}{2}$ or $ABC = \pi - (2.2395)$ oe	M1	For attempt to find angle AOC or ABC $AOC = 2\pi - 2(\textit{their } AOB)$ $ABC = \pi - (\textit{their } AOB)$ oe
	$AOC = 1.804$ or 1.803	A1	Condone 1.8 or 1.80
	Arc length = 18.04 or 18.03	M1	For attempt at arc length using $10 \times \textit{their } AOC$
	$AC = 20\sin\frac{AOC}{2}$ or $36\sin\frac{ABC}{2}$ or $\sqrt{10^2 + 10^2 - 200\cos AOC}$ or $\sqrt{18^2 + 18^2 - 648\cos ABC}$ $= 15.69$ or 15.7	M1	For attempt at AC using *their* AOC, or ABC but $AOC \neq 2.24$ or $\frac{2\pi}{3}$
	Perimeter = 33.7	A1	Allow awrt 33.7

32)

(i)	$100 = 2r + 2r\theta + 3r\theta$	M1	for addition of $2r$ and two arc lengths with at least one correct arc length
	$\theta = \frac{100 - 2r}{5r}$ or $\frac{20}{r} - \frac{2}{5}$ oe	A1	
(ii)	$\frac{1}{2}9r^2\theta - \frac{1}{2}4r^2\theta$	M1	for subtraction of two sector areas with at least one sector area correct.
	$\frac{5r^2}{2}\left(\frac{100-2r}{5r}\right)$	A1	Must expand and simplify to obtain given answer $50r - r^2$
(iii)	$\frac{dA}{dr} = 50 - 2r$ $0 = 50 - 2r$ leading to $r = 25$	M1	for differentiation and equating to zero and obtaining r or for using completing the square $-(25-r)^2 + 25^2$
	Max when $A = 625$	A1	

(iv)	When $r = 10$, $\frac{dA}{dr} = 30$	B1	
	$\frac{dr}{dt} = \frac{3}{30}$	M1	for $\frac{dr}{dt} = \frac{3}{their 30}$ where *their* 30 has been obtained from an evaluation of $\frac{dA}{dr}$ at $r = 10$
	$\frac{dr}{dt} = 0.1$ or $\frac{1}{10}$	A1	
(v)	$\frac{d\theta}{dr} = -\frac{20}{r^2}$ oe	B1	
	$\frac{d\theta}{dr} = -\frac{1}{5}$ oe $\frac{d\theta}{dt} = \frac{1}{10} \times -\frac{1}{5}$ oe	M1	for *their* $\frac{dr}{dt} \times$ *their* $\frac{d\theta}{dr}$ with both evaluated at $r = 10$
	$\frac{d\theta}{dt} = -\frac{1}{50}$ or -0.02	A1	

33)

(i)	**Either** $15^2 = 10^2 + 10^2 - 200\cos AOB$ $\cos AOB = -0.125$	M1	For use of cosine rule
	$AOB = 1.696$ so 1.70 to 2 dp	A1	Must have justification to 2 dp
	Or $\sin\left(\frac{AOB}{2}\right) = \frac{7.5}{10}$ $\frac{AOB}{2} = 0.8481$	M1	For use of basic trig
	$AOB = 1.696$ so 1.70 to 2 dp	A1	

P.T.O

(ii)	Angle $DOC = \frac{\pi}{3}$	B1	
	Either $AOD = BOC = 0.5\left(2\pi - \frac{\pi}{3} - 1.696\right)$ $AOD = BOC = 1.77$	M1	For attempt to get AOD or BOC
	Arc lengths = 17.7	M1	For attempt at arc length using their previous answer
	Perimeter = $15 + 10 + (2 \times 17.7) = 60.4$	A1	
	Or Arc $AB = 17$ or Arc $CD = \frac{10\pi}{3}$	M1	For either arc length
	$(20\pi - \text{arc } AB - \text{arc } CD)$	M1	
	Perimeter = 60.4	A1	
(iii)	**Either** Area of each sector = $\frac{1}{2}10^2(1.770)$	M1	For area of sector using their BOC
	Area of triangles = $\left(\frac{1}{2} \times 100 \times \sin\frac{\pi}{3}\right) + \left(\frac{1}{2} \times 100 \sin 1.70\right)$	M1	For area of one triangle using the sine rule oe
	Total area = $177 + 43.3 + 49.6$	M1	For plan
	Area = awrt 270	A1	
	Or Area of upper segment = $\frac{1}{2}10^2(1.696 - \sin 1.696)$	M1	For area of a sector or area of a triangle using the sine rule oe
	Area of lower segment = $\frac{1}{2}10^2\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$	M1	For whichever has not been obtained in previous part
	Shaded area = 100π – are of the 2 segments Area = $314.2 - 35.2 - 9.06$	M1	For plan
	Area = awrt 270	A1	

P.T.O 34)

(i)	[angle ECD =] $\frac{5\pi}{18}$ oe or 0.873 soi	**B1**	
	Attempts to find AC and subtract 8	**M1**	e.g. $AC = \frac{8}{\cos\frac{2\pi}{9}}$
	$[DC =]\ 2.44$	**A1**	
	$\frac{1}{2} \times 8 \times their AC \times \sin\frac{2\pi}{9}$ OR $\frac{1}{2} \times 8 \times 8\tan\left(\frac{2\pi}{9}\right) - \frac{1}{2} \times 8^2 \times \frac{2\pi}{9}$ $- \frac{1}{2} \times their\, 2.44^2 \times their \frac{5\pi}{18}$	**M2**	**M1** for $\frac{1}{2} \times 8^2 \times \frac{2\pi}{9}$ or for $\frac{1}{2} \times their\, 2.44^2 \times their \frac{5\pi}{18}$ seen
	awrt 1.91	**A1**	
(ii)	$their(6.712 - 2.443)$ $+ their\ 2.443\left(\frac{5\pi}{18}\right) + 8\left(\frac{2\pi}{9}\right)$	**M2**	**M1** for either arc seen
	awrt 12.0	**A1**	

35)

(i)	$[AD = BC =]\ 35$ soi	**B1**	
	Valid method for finding DC	**M1**	
	$[DC =]\ 19.2836...$	**A1**	
	$50 \times \frac{4\pi}{9}$ oe	**M1**	
	$35 + 35 + 19.2836\ldots + 50 \times \frac{4\pi}{9}$ = 159 or awrt 159 isw	**A1**	
(ii)	Sector – triangle: $\frac{1}{2} \times 50^2 \times \frac{4\pi}{9}$	**M1**	or Segment + trapezium : $\frac{1}{2} \times 50^2\left(\frac{4\pi}{9} - \sin\frac{4\pi}{9}\right)$
	$-\left(\frac{1}{2} \times their 15^2 \times \sin\left(\frac{4\pi}{9}\right)\right)$ oe	**M1**	$+\left(\frac{1}{2}(64.2787... + 19.2836...) \times 26.81155\right)$
	1630 or 1634.538… rot to 4 or more figs, isw	**A1**	

36)

(i)	$\frac{1}{2}r^2\theta = 48$	B1	
	$\theta = \frac{96}{r^2}$	B1	**Dep** Must have previous B1
(ii)	$r\theta = 12$	B1	For statement of arc length
	$r \times \frac{96}{r^2} = 12$	M1	For attempt to use part **(i)** to find either r or θ
	$r = 8$ and $\theta = 1.5$	A1	For both
(iii)	Area = $48 - \left(\frac{1}{2}r^2 \sin\theta\right)$	M1	For a complete method including a correct attempt for the area of the triangle using their r and θ
	=16.1	A1	

37)

(i)	(Arc length =) 1.5×5 oe soi	M1	implied by 7.5
	(DE =) $10\sin(0.75)$ oe soi	M1	implied by awrt 6.82
	34.3 or answer in range 34.31 to 34.32	A1	
(ii)	(Area sector =) $\frac{1}{2} \times 5^2 \times 1.5$ oe	M1	implied by 18.75
	(Area triangle =) $\frac{1}{2} \times 5^2 \times \sin(1.5)$ oe	M1	implied by awrt 12.47
	31.2 or answer in range 31.21 to 31.22	A1	

38)

(i)	$16x = 40$ oe	M1	
	$x = 2.5$ oe (radians)	A1	
(ii)	$\frac{1}{2}(16)^2(2.5)$ oe	M1	
	320	A1	
(iii)	$\frac{1}{2}r^2(their\,2.5) = (their\,320) - 140$ oe	M1	FT provided *their* 320 > 140
	correct simplification to $r^2 = \ldots$	M1	**dep** on first **M1**
	12	A1	

39)

(i)	1.48	B1
(ii)	$\frac{1}{2}\times 10^2 \times \theta = 21.8$	M1
	$\theta = 0.436$	A1
(iii)	$\angle BOC = \frac{2\pi - 1.48 - 0.436}{2}$ $(= 2.18(4))$	B1
	$BC = 20\sin\left(\frac{1}{2}\angle BOC\right)$ or $BC = \frac{10\times \sin BOC}{\sin\left(\frac{\pi - BOC}{2}\right)}$ or $BC = \sqrt{(200 - 200\cos BOC}$ $BC = 17.7(5)$	M2
	Perimeter = $14.8 + (2\times 17.7(5)) + 4.36$ $= 54.7$ or 54.6	A1
(iv)	Area = $\left(\frac{1}{2}\times 10^2 \times 1.48\right) + 21.8 + 2\left(\frac{1}{2}\times 10^2 \sin 2.18(4)\right)$	B2
	$= 178$	B1
	<u>Alternative method 1</u> Segment area = $\frac{1}{2}\left(10^2\left(2.18 - \sin 2.18\right)\right)$	B1
	Area required = $100\pi - 2\times\frac{1}{2}\left(10^2\left(2.18(4) - \sin 2.18(4)\right)\right)$	B1
	$= 178$	B1
	<u>Alternative method 2</u> Area of trapezium = $\frac{1}{2}\left((13.5 + 4.33)(17.1)\right)$	B1
	Area of segments = $\frac{1}{2}\left(10^2\left(1.48 - \sin 1.48\right)\right)$ + $\frac{1}{2}\left(10^2\left(0.436 - \sin 0.436\right)\right)$	B1
	$= 178$	B1

40)

(i)	$5\angle BAC = 6.2$, $\angle BAC = 1.24$	B1
(ii)	$\sin 0.62 = \frac{BD}{5}$, $BD = 2.905$, 2.91	B1
	Arc BFC: $\pi \times BD \quad (= 9.13)$	M1
	Perimeter: $9.13 + 6.2 = 15.3$	A1
(iii)	Area: $\left(\frac{1}{2} \times \pi \times 2.91^2\right) - \left(\left(\frac{1}{2} \times 5^2 \times 1.24\right) - \left(\frac{1}{2} \times 5^2 \times \sin 1.24\right)\right)$	B3
	$9.58 \leqslant$ Area $\leqslant 9.62$	B1

41)

(i)	$\tan\left(\frac{PAQ}{2}\right) = 2.4$	M1
	$PAQ = 2.352(01....)$ $PAQ = 2.35$ correct to 3 sf	A1
(ii)	$PBQ = 0.790$ or 0.792	B1
(iii)	$(2.352 \times 10) + (0.790 \times 24)$	M1,A1
	= awrt 42.5	A1
(iv)	$\left(\left(\frac{1}{2} \times 24^2 \times 0.790\right) - \left(\frac{1}{2} \times 24^2 \times \sin 0.790\right)\right)$	B1,B1
	$+\left(\left(\frac{1}{2} \times 10^2 \times 2.352\right) - \left(\frac{1}{2} \times 10^2 \times \sin 2.352\right)\right)$	B1
	$= 22.94 + 82.1$ $= 105$	B1

42)

(i)	$\frac{\pi}{3}$
	6 [cm]
(ii)	[major arc =] $\left(2\pi - their\frac{\pi}{3}\right) their\ r$
	$10\pi + 6$ cao
(iii)	$\frac{1}{2}(their 6)^2\left(2\pi - their\frac{\pi}{3}\right)$
	$\frac{1}{2}(their 6)^2 \sin\left(their\frac{\pi}{3}\right)$
	Sector + triangle
	$30\pi + 9\sqrt{3}$

P.T.O **43)**

(i)	0.5
(ii)	$15^2 = 8^2 + 8^2 - (2 \times 8 \times 8 \times \cos AOB)$ $AOB = 2.43075$ rads
	$DOC = AOB - 2(\textit{their } AOD)$
	$DOC = 1.43$ to 2 dp
	Alternative 1 $15 = 2 \times 8 \times \sin\left(\frac{1 + DOC}{2}\right)$
	use of $\frac{1 + 0.5DOC}{2}$
	$DOC = 1.43$ to 2 dp
	Alternative 2
	$15^2 = 8^2 + 8^2 - (2 \times 8 \times 8 \times \cos AOB)$ $AOB = 2.43075$ rads $\angle AOB \times 8 =$ arc AB
	$\frac{\text{arc}AB - 8}{8} = \angle DOC$
	$DOC = 1.43$ to 2 dp
	Alternative 3 Equating 2 different forms for the area of triangle AOB $\frac{15\sqrt{31}}{4} = \frac{1}{2} \times 8^2 \sin AOB$, $AOB = 2.43075$ rads
	$DOC = AOB - 2(\text{their } AOD)$
	$DOC = 1.43$ to 2 dp

44)

Mark both parts of this question together	
$18\pi = \theta r$	B1
$126\pi = \frac{1}{2}\theta r^2 \quad (\Rightarrow 252\pi = \theta r^2)$	B1
$\frac{252\pi}{18\pi} = \frac{\theta r^2}{\theta r} \Rightarrow 14 = r$	M1A1
$18\pi = \theta \times 14 \Rightarrow \theta = \frac{9\pi}{7}$ oe	A1
	(5)
ALT	
$A = \frac{1}{2}rl \Rightarrow 126\pi = \frac{1}{2} \times r \times 18\pi \Rightarrow r = 14$	M1A1A1
$18\pi = 14\theta \Rightarrow \theta = \frac{9}{7}\pi$	B1B1
	(5)

45)

(a)	$S = \frac{1}{2} \times 10^2\theta - \frac{1}{2} \times 6^2\theta$
	$= 32\theta$ *
(b)	$\frac{dS}{dt} = 32\frac{d\theta}{dt}$
	$\frac{dS}{dt} = 32 \times 0.2 = 6.4$
(c)	$20 = 32\theta$
	$\theta = \frac{20}{32}$ oe inc 0.625
	Perim $= 10 \times \frac{20}{32} + 6 \times \frac{20}{32} + 2 \times 4$
	$= 18$ cm
	ALT: Perim $= 10\theta + 6\theta + 8 = 16\theta + 8 = \frac{1}{2}S + 8 = 18$