Chapter 2 - Functions

Types: one to one
many to one

can be asked to find:
Domain and Range

Composite function are 2 functions linked together.
eg $f^2 = ff(x)$

inverse function
↳ only exists for one to one

The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are reflections of each other in the line $y = x$

↳ Domain of $f(x)$ becomes range of $f^{-1}(x)$

↳ Range of $f(x)$ becomes Domain of $f^{-1}(x)$

Absolute value function
↳ To draw the graph of $y = |f(x)|$, first draw the graph of $y = f(x)$ then reflect any -ve parts in the x-axis

Chapter 3 - Quadratic functions

$f(x) = ax^2 + bx + c$
if $a > 0$ then min value
if $a < 0$ then max value

$f(x) = a(x-h)^2 + k$
when $x = h$
if $a > 0$ the min value at k
if $a < 0$ then max value at k

To find max/min value
↳ use completing square

To sketch a graph find:
- The shape of curve, ∪ or ∩
- turning point
- where it cuts y axis → $f(0)$

To find root type
↳ $D = b^2 - 4ac$

If $D > 0$ → 2 diff real roots
$D = 0$ → 2 equal real roots
$D < 0$ → no real roots

Quadratic inequalities
$f(x) > 0$
↳ asking for points above x axis
$f(x) < 0$
↳ asking for points below x axis

Chapter 4 - Indices and surds

Indices:

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^m \times b^m = (ab)^m$

$a^m \div b^m = \left(\frac{a}{b}\right)^m$

$a^0 = 1$

$a^{-n} = \frac{1}{a^n}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Surds:

$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$

$\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

Chapter 5 - Simultaneous equations

To solve one linear and one non-linear simultaneous equations
1) use substitution method
2) obtain quadratic equation and solve
3) use corresponding values for second variable

Simultaneous can be used to
↳ find the point of intersections of a line and a curve.

To solve practical problems involving a linear and a non-linear equation, first formulate the equations and then solve them using simultaneous method.

Chapter 6 — factors of polynomials

If polynomial $f(x)$ is divided by $(x-a)$ then $f(a)$ is remainder

If $x-a$ is a factor of $f(x)$ then $f(a)=0$

Factorisation

find first factor using trial and error then use

1) Long division method

or

2) Inspection method

↓

$(x+p)(Ax^2+Bx+C)$
(factor)

Solving cubic equation
↳ factorise
↳ if $x-a$ is a factor of $f(x)$ then $x=a$ is root of $f(x)$

Chapter 7 — Logarithmic and exponential functions

$y=a^x \longleftrightarrow \log_a y = x$

$\log_a PQ = \log_a P + \log_a Q$

$\log_a \frac{P}{Q} = \log_a P - \log_a Q$

$\log_a P^n = n \log_a P$

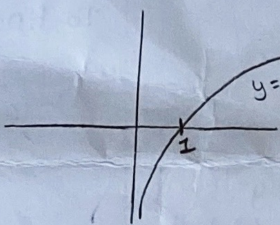
$\log_a 1 = 0$

$\log_a a = 1$

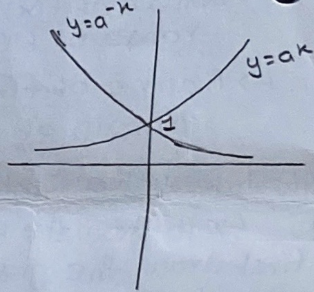
Changing base
↳ $\log_b a = \frac{\log_c a}{\log_c b}$

$\log_{10} x = \lg x$

$\log_e x = \ln x$



log graph cuts x axis at 1



exponential graph cuts y axis at 1

Chapter 8 - Straight line graphs

$Y = mX + C$

Distance $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

parallel lines → $m_1 = m_2$

perpendicular lines $m_1 m_2 = -1$

gradient $(m) = \frac{\text{rise}}{\text{run}} = \frac{y_2-y_1}{x_2-x_1}$

Chapter 9 - Circular measures

$1° \rightarrow \frac{\pi}{180}$

$1 \text{ rad} \rightarrow \frac{180}{\pi}$

$S = r\theta$

Area of sector $= \frac{1}{2}r^2\theta$

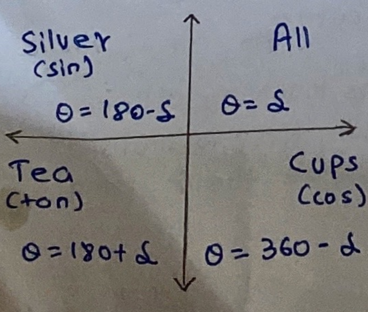
↳ always calculate θ in radians

Chapter 10 — trignometry

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$



$\frac{1}{\sin\theta} = \operatorname{cosec}\theta$

$\frac{1}{\cos\theta} = \sec\theta$

$\frac{1}{\tan\theta} = \cot\theta$

Imp to know graphs of:

sin → period 360
cos → period 360
tan → period 180

→ period $= \frac{360/180}{b}$

max/min $= c+a / c-a$

Identities

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\sin 2\theta = 2\sin\theta \times \cos\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

Chapter 11 — Combinations and Permutations

combination — selection

$$^nC_r = \frac{n!}{r!(n-r)!}$$

Permutation — arrangement

$$^nP_r = \frac{n!}{(n-r)!}$$

$$^nC_r = \frac{^nP_r}{r!}$$

$$^nC_r = {^nC_{n-r}}$$

Chapter 12 — Binomial expansions

→ Pascal's triangle

$$T_{r+1} = {^nC_r} \times a^{n-r} \times b^r$$

Extention — Arithmatic and Geometric Progression

AP

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(a+l)$$

$$= \frac{n}{2}(2a + (n-1)d)$$

GP

$$a_n = a \cdot r^{n-1}$$

If $r = 1$.

$$S_n = an$$

If $r > 1$

$$S_n = \frac{a(r^n - 1)}{(r-1)}$$

If $r < 1$

$$S_n = \frac{a(1-r^n)}{1-r} \qquad S_\infty = \frac{a}{1-r}$$

Chapter 13 — Vectors

- Position vector
  ↳ taken from origin

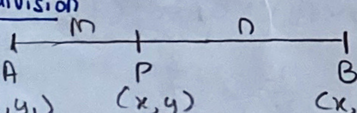
If P, Q, R are collinear then

$$\vec{PQ} = m \cdot \vec{QR}$$

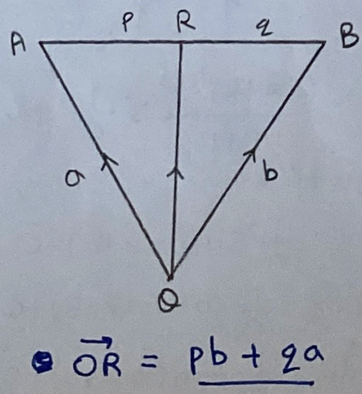
If $mp + nq \parallel ap + bq$

then $\frac{m}{a} = \frac{n}{b}$

internal division

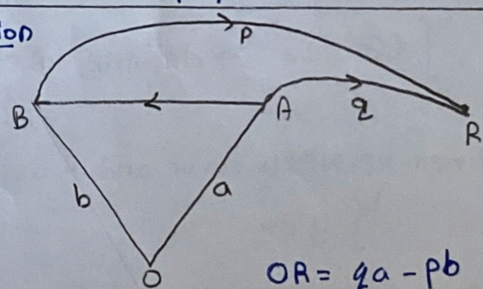


- $$P(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

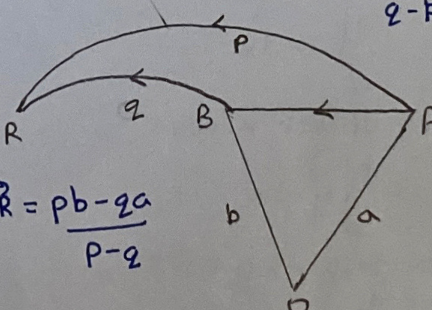


- $$\vec{OR} = \frac{pb + qa}{p+q}$$

external division



$$OR = \frac{qa - pb}{q-p}$$



$$\vec{OR} = \frac{pb - qa}{p-q}$$

$$V_{A/B} = V_A - V_B$$

PV of A + Velocity vector $\times t$ = PV of B + VV $\times t$

PV at t = (VV $\times t$) + initial PV

if a and b are perpendicular

↳ $(a_1 i + b_1 j) + (a_2 i + b_2 j) = a_1 a_2 + b_1 b_2$

$$\vec{a} = |\vec{a}| = a$$

$$\vec{r} = xi + yj$$

$$|\vec{r}| = \sqrt{x^2 + y^2}$$

unit vector $\hat{r} = \frac{xi + yj}{\sqrt{x^2+y^2}}$

Chapter 15 - differentiation

gradient = $\frac{dy}{dx}$

If $y = ax^n$

$\frac{dy}{dx} = nax^{n-1}$

Product rule = $uv' + vu'$

Quotient rule = $\frac{vu' - uv'}{v^2}$

$$\delta y = \left(\frac{dy}{dx}\right)_{x=k} \times \delta x$$

where δx = new − old

% change = $\frac{\delta y}{y} \times 100$

Chapter 16 - Applications of Differentiation

gradient of tangent = $\frac{dy}{dx}$

gradient of normal = $\frac{-1}{\frac{dy}{dx}}$

Stationary point

↳ $\frac{dy}{dx} = 0$

If $\frac{d^2y}{dx^2} > 0$ then curve has min point

$\frac{d^2y}{dx^2} < 0$ then curve has max point

$\frac{d^2y}{dx^2} = 0$ then point of inflexion

Chapter 17 - Integration

If $\frac{dy}{dx} = f(x)$ then $y = \int f(x)\,dx + c$

If $\frac{dy}{dx} = ax^n$ then $y = \frac{ax^{n+1}}{n+1}$

$\int_a^b f(x)\,dx$ → definite integration

Area between curve and x axis

$\int_a^b y\,dx$

Area between curve and y axis

$\int_c^d x\,dx$

Area under x axis

$-\int_a^b y \cdot dx$

Area left of y axis

$-\int_c^d x \cdot dx$

Area between 2 curves/line

$\int_a^b [f(x) - g(x)]\,dx$

where $f(x) > g(x)$

Chapter 18 - Further Differentiation and Integration

Differentiation

$\frac{d}{dx}\sin(ax+b) = a\cos(ax+b)$

$\frac{d}{dx}\cos(ax+b) = -a\sin(ax+b)$

$\frac{d}{dx}\tan(ax+b) = a\sec^2(ax+b)$

$\log x \cdot dx = \frac{1}{x}$

$\ln f(x) \cdot dx = \frac{1}{f(x)} \times f'(x)$

$e^{f(x)} \cdot dx = e^{f(x)} \times f'(x)$

Integration

$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{n+1} \times \frac{1}{a} + c$

$\int \frac{1}{x} \cdot dx = \log x + c$

$\int \cos(ax+b) \cdot dx = \frac{1}{a}\sin(ax+b) + c$

$\int \sin(ax+b) \cdot dx = -\frac{1}{a}\cos(ax+b) + c$

$\int \sec^2(ax+b) \cdot dx = \frac{1}{a}\tan(ax+b) + c$

$\int e^x \cdot dx = e^x + c$

$\int e^{ax+b} \cdot dx = e^{ax+b} \times \frac{1}{a} + c$

...apter 18 – Kinematics

velocity $v = \frac{ds}{dt}$

acceleration $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

$v = \int a \cdot dt$

$s = \int v \cdot dt$

instantaneous rest
↳ $v = 0$

negative acceleration
↳ deceleration

In s-t graph
gradient $= v$

In v-t graph
gradient $= a$

equations for constant a:

$v = u + at$

$s = \frac{u+v}{2} t$

$s = ut + \frac{1}{2} at^2$

$v^2 = u^2 + 2as$